LECTURE 7

Technology

5SSPP217 – Intermediate Microeconomics

Recap: contingent consumption and expected utility

A **contingent consumption plan** specifies consumption in every state of the world.

Example

- One plan is purchasing a lottery ticket which yields £95 in the bad state and £295 in the good state;
- another plan is not buying a lottery ticket which yields £100 in either state.

An individual's expected utility is given by the weighted sum of utility in each state, where the weights are the probability of that state occurring:

$$U(c_b, c_g) = \pi u(c_b) + (1 - \pi)u(c_g).$$

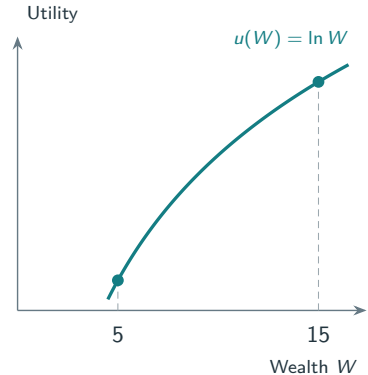
Recap: risk aversion

Suppose a gamble returns £5 with 50% probability and £15 with 50%, then $E[W] = 10$

A **risk-averse** consumer has concave utility and prefers a certain amount of money to a gamble with the same mean amount:

$$u(E[W]) \geq E[u(W)].$$

In this example: a certain £10 is preferred to an equal chance of £5 or £15.



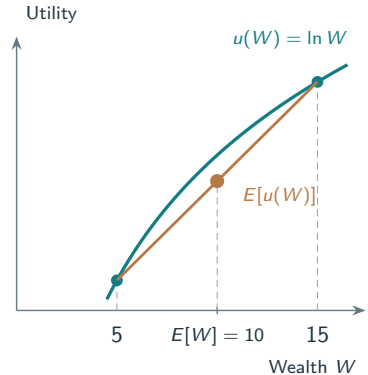
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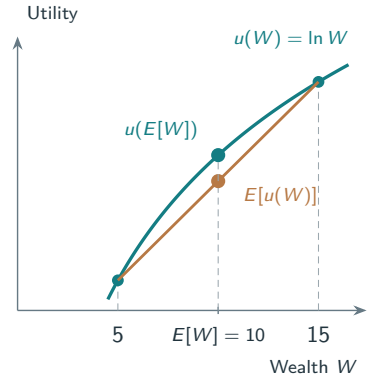
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Recap: Insurance

In order to reduce the riskiness of their consumption, individuals can buy $£K$ of insurance coverage at a price of γK.

With insurance, consumption in the good and the bad state is given by

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Then, a risk-averse consumer buys **full coverage**:

$$K^* = L, \qquad c_b = c_g = m - \pi L.$$

Technology and firm behaviour

In today's lecture we move from consumer choice to firm behaviour.

Economists use the word **technology** to describe how inputs can be turned into outputs and this will be the main constraint on firm behaviour.

Inputs, outputs and factors of production

Inputs are also called the **factors of production** and are often classified in broad categories such as labour, capital, land, and raw materials.

The word "capital" is often used to refer to two related but different concepts:

- **Financial capital** is the funding obtained from owners, lenders or past profits a firm uses to acquire productive assets and finance its operations.
- **Physical capital** consists of produced inputs such as machines, buildings and computers, used to produce other goods and services.

Inputs and outputs are usually measured in *flow* units: a certain amount of units per period, e.g., 10 hours of labour per week.

Production sets and production functions

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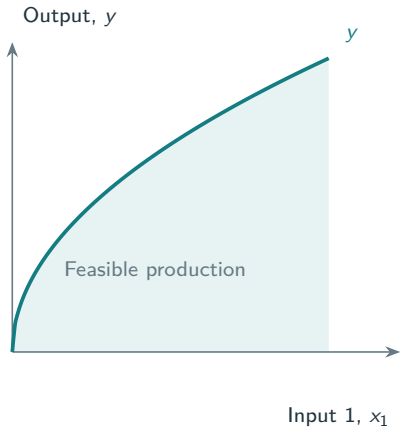
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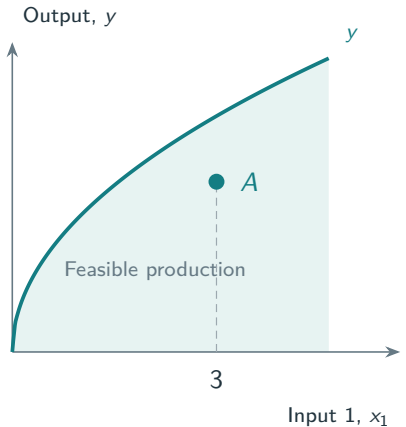
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A is feasible but below the frontier.



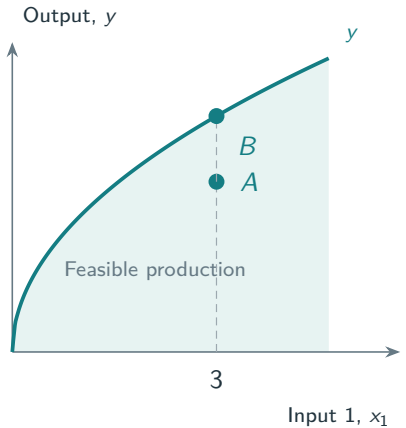
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A is feasible but below the frontier.
B gives maximum output.



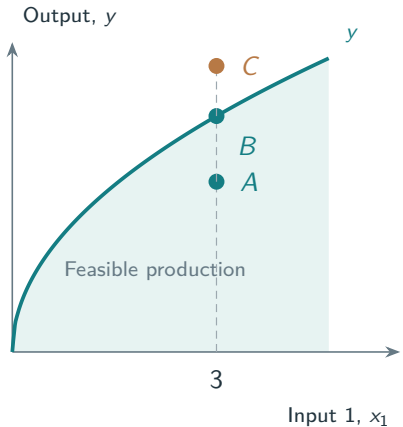
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Suppose $y = \sqrt{x_1 x_2}$ and $x_2 = 4$:
A is feasible but below the frontier.
B gives maximum output.
C is infeasible.



Marginal products

Marginal products

The **marginal product** is equal to the additional amount of output if we increase an input by one unit, holding all other inputs fixed.

$$MP_1 = \frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}.$$

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We often express the marginal product as the partial derivative of the production function w.r.t. one of the inputs:

$$MP_1(x_1, x_2) = \frac{\partial f}{\partial x_1}, \qquad MP_2(x_1, x_2) = \frac{\partial f}{\partial x_2}.$$

Cobb–Douglas marginal products

Suppose the production function is Cobb–Douglas, the marginal products are then obtained by differentiating $y = f(x_1, x_2) = Ax_1^a x_2^b$:

$$MP_1 = Ax_2^b \frac{\partial x_1^a}{\partial x_1} = aAx_1^{a-1}x_2^b,$$

$$MP_2 = bAx_1^a x_2^{b-1}.$$

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In our example A=1, $a = b = 1/2$, $f(x_1, x_2) = \sqrt{x_1 x_2}$:

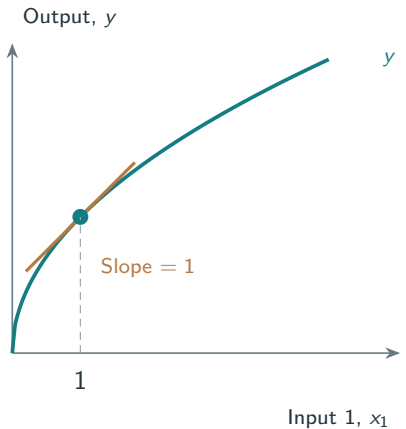
$$MP_1 = \frac{1}{2}\sqrt{\frac{x_2}{x_1}}, \qquad MP_2 = \frac{1}{2}\sqrt{\frac{x_1}{x_2}}.$$

Diminishing marginal product

Often, as we add additional units of one input while holding other inputs fixed, output will increase at a slower and slower rate: the "law" of **diminishing marginal product**.

For $f = \sqrt{x_1 x_2}$ with $x_2 = 4$:

$$y = 2\sqrt{x_1}, \qquad MP_1 = \frac{1}{\sqrt{x_1}}.$$

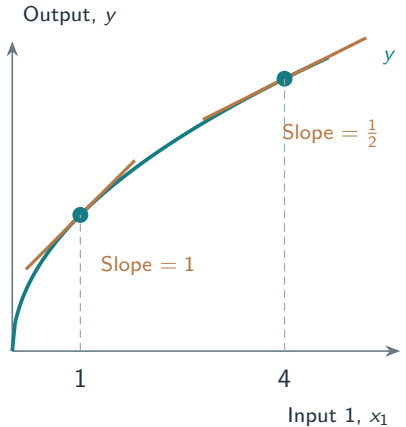


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Isoquants and examples of technology

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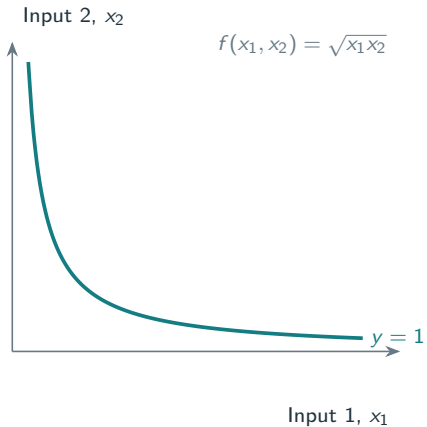
Iso- comes from the Greek *isos*, meaning "equal".

An **isoquant** shows all the combinations of two inputs x_1 and x_2 that can produce the same amount of output $\bar{y}$

$$f(x_1, x_2) = \bar{y}.$$

One can easily obtain the expression for the isoquant curve. Suppose $y = \sqrt{x_1 x_2}$, then

$$\bar{y}^2 / x_1 = x_2$$



Isoquants

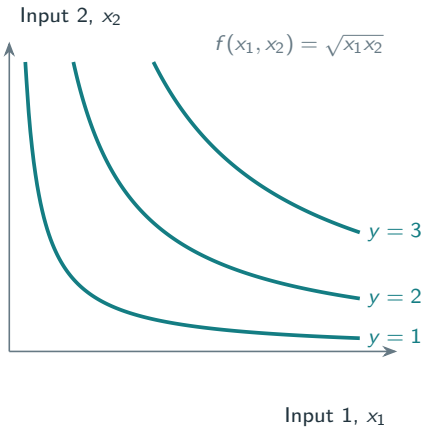
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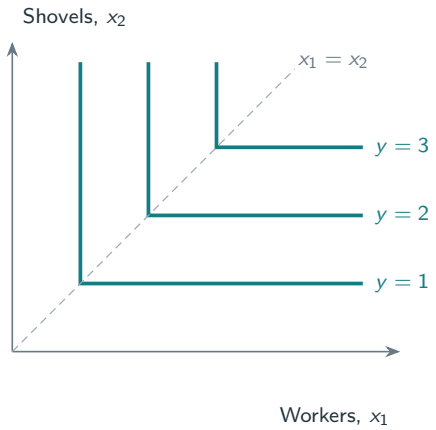


Fixed proportions

Under this technology, output is produced by combining inputs in fixed proportions.

For example, suppose a firm produces holes and each hole requires one worker and one shovel.

$$y = f(x_1, x_2) = \min\{x_1, x_2\}.$$



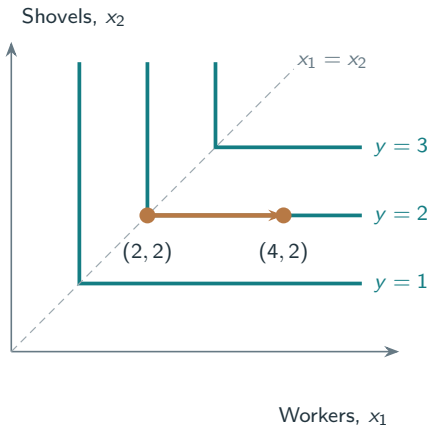
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Increasing the number of workers from 2 to 4 without increasing shovels has no effect on output.

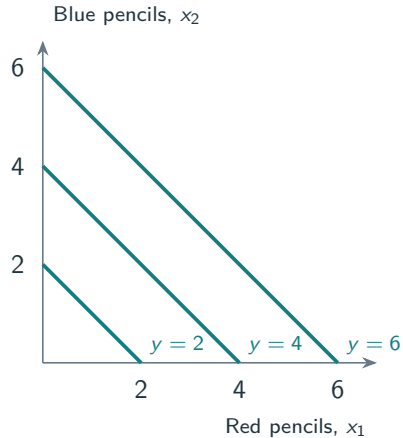


Perfect substitutes

Under this technology, one input can replace another at a constant rate.

For example, suppose red and blue pencils are equally useful for producing homework.

$$y = f(x_1, x_2) = x_1 + x_2.$$



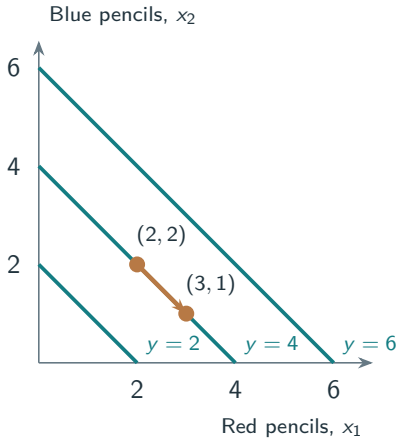
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Replacing one blue pencil with one red pencil moves us from $(2, 2)$ to $(3, 1)$ without changing output.



Cobb–Douglas technology

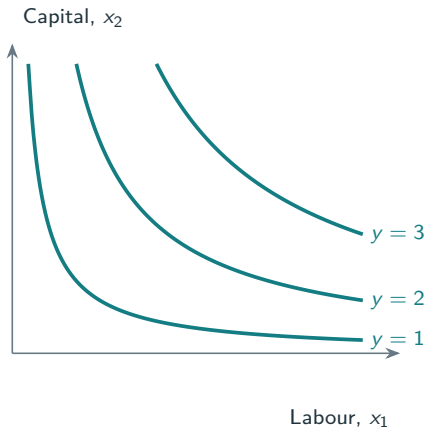
With labour x_1 and capital x_2, this technology has the form:

$$y = f(x_1, x_2) = A x_1^a x_2^b,$$

where $A > 0$ and $a, b > 0$.

For example, suppose $A = 1$ and $a = b = \frac{1}{2}$:

$$y = f(x_1, x_2) = \sqrt{x_1 x_2}.$$



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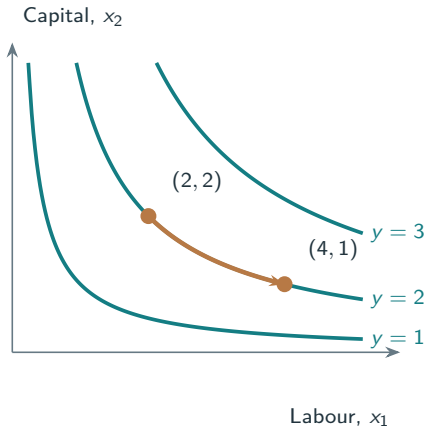
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Doubling labour from 2 to 4 allows us to halve capital from 2 to 1 without changing output.



Properties of technology

Monotonicity and convexity

We commonly make two assumptions about technologies.

Monotonicity: adding more inputs cannot reduce maximum output.
If $x_1' \geq x_1$ and $x_2' \geq x_2$, then

$$f(x_1', x_2') \geq f(x_1, x_2).$$

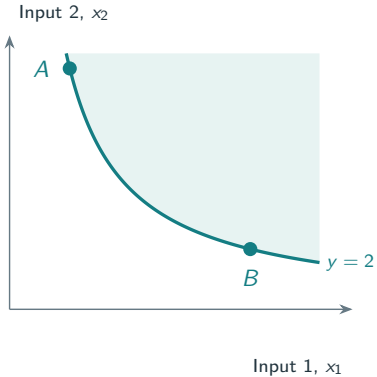
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Convexity: if two input bundles can produce a certain level of output $\bar{y}$, any weighted average of them can produce at least $\bar{y}$.



Monotonicity and convexity

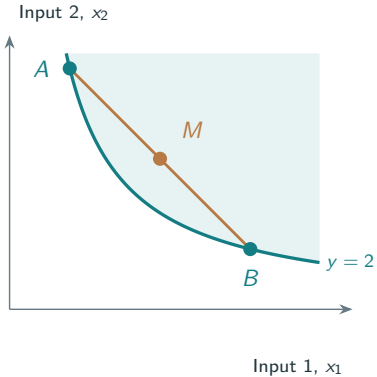
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A and B each produce 2. Their midpoint M can produce at least 2.



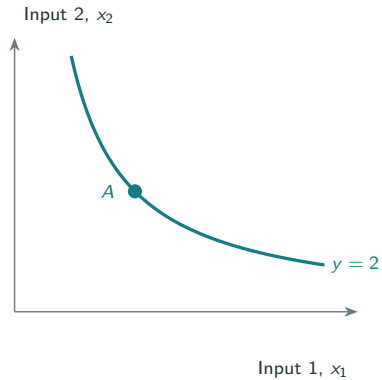
The technical rate of substitution

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The **technical rate of substitution** measures by how much input 2 can fall if we increase input 1 by a small amount while keeping output fixed.

Suppose we change x_1 and x_2 such that y remains fixed:

$$dy = \frac{\partial f}{\partial x_1}\, dx_1 + \frac{\partial f}{\partial x_2}\, dx_2 = 0.$$

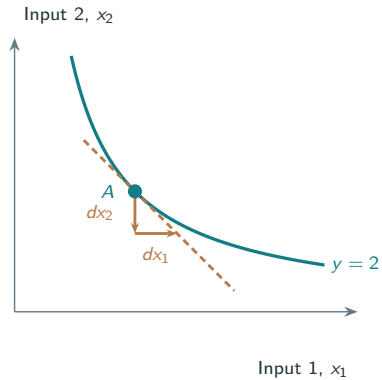


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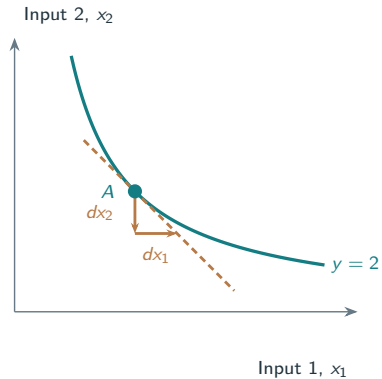
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$$dy = \frac{\partial f}{\partial x_1}\, dx_1 + \frac{\partial f}{\partial x_2}\, dx_2 = 0.$$

With $\partial f / \partial x_2 > 0$, rearranging gives

$$TRS = \left.\frac{dx_2}{dx_1}\right|_y = -\frac{\partial f / \partial x_1}{\partial f / \partial x_2}.$$

The absolute value of the TRS is equal to the ratio of marginal products

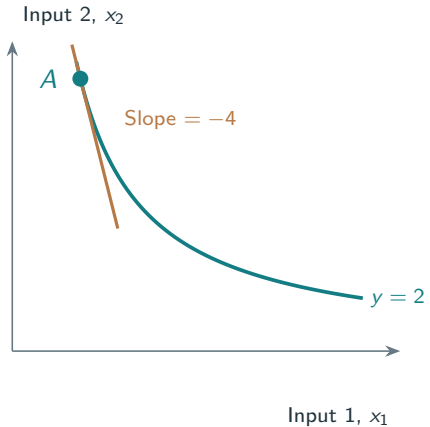


Diminishing technical rate of substitution

We often assume that the absolute value of the technical rate of substitution declines as we increase x_1.

As we add more x_1, we need increasingly large amounts of x_1 to compensate for a fall in x_2.

Note that this is a statement about the ratio of marginal products, not about the marginal products themselves.

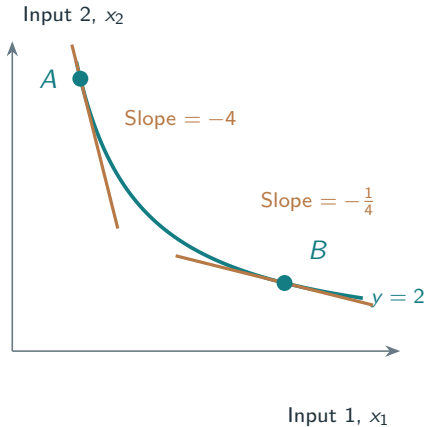


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Returns to scale

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Returns to scale describe how output responds when all inputs scale proportionately.

Suppose we multiply all inputs by $t > 1$, does this increase output by more or less than t times?

Constant returns $f(tx_1, tx_2) = tf(x_1, x_2)$

Increasing returns $f(tx_1, tx_2) > tf(x_1, x_2)$

Decreasing returns $f(tx_1, tx_2) < tf(x_1, x_2)$

Cobb–Douglas and returns to scale

Suppose the production function is Cobb-Douglas, $f(x_1, x_2) = Ax_1^a x_2^b$, then scaling both inputs gives

$$\begin{aligned} f(tx_1, tx_2) &= A(tx_1)^a (tx_2)^b \\ &= t^a t^b A x_1^a x_2^b \\ &= t^{a+b} f(x_1, x_2). \end{aligned}$$

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Compare the output multiplier t^{a+b} with t:

$$\begin{aligned} a + b = 1 &\implies \text{constant returns,} \\ a + b > 1 &\implies \text{increasing returns,} \\ a + b < 1 &\implies \text{decreasing returns.} \end{aligned}$$

The short run and the long run

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Economists define the **short run** as the period for which at least one input is fixed.

If input 2 is fixed at $\bar{x}_2$:

$$y = f(x_1, \bar{x}_2).$$

In the **long run**, all inputs can be varied.

Review

We have introduced several related but distinct concepts:

Marginal product	Change one input, holding the others fixed.
TRS	Change the input mix, holding output fixed.
Returns to scale	Scale all inputs in the same proportion.

Next lecture: Profit maximisation. What is the optimal production plan given technology and prices?