LECTURE 6

Uncertainty

5SSPP217 – Intermediate Microeconomics

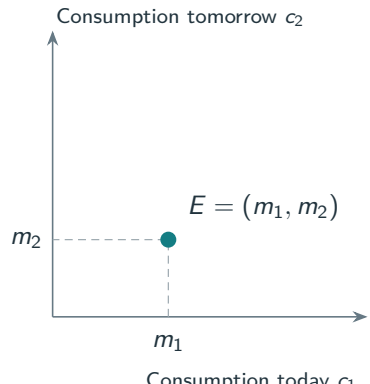
Recap: consumption over time

Intertemporal choice is the choice of how much to consume over time.

A consumer with endowments (m_1, m_2) chooses c_1 and c_2 subject to his budget constraint:

$$\max_{c_1, c_2 \geq 0} \; U(c_1, c_2)$$

$$\text{s.t.} \quad c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}.$$



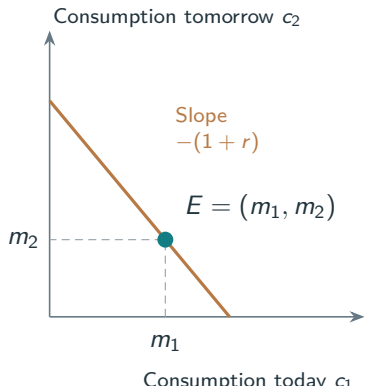
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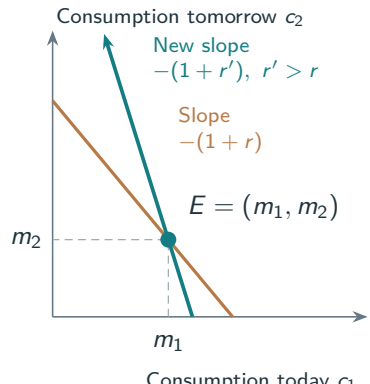
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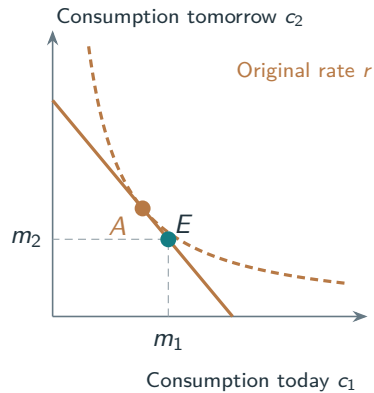
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Recap: saving, borrowing and interest rates

A consumer may choose to **save** ($c_1 < m_1$) or **borrow** ($c_1 > m_1$).



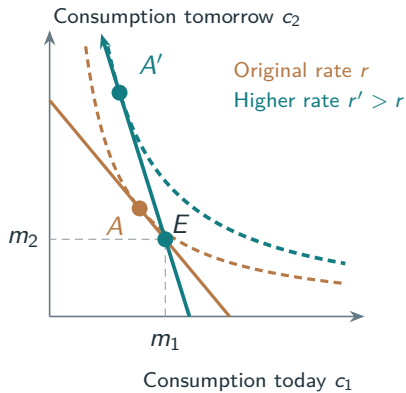
Recap: saving, borrowing and interest rates

A consumer may choose to **save** ($c_1 < m_1$) or **borrow** ($c_1 > m_1$).

If current consumption is a normal good, a rise in the interest rate gives rise to:

- **substitution effect**: consume less today;
- **income effect**: consume more today as a saver, less as a borrower.

The total effect is ambiguous if the income and substitution effects have opposite signs.



Today: choice under uncertainty

Many choices involve considerable uncertainty: is it worth studying economics at KCL?

In today's lecture, we study how consumers choose when outcomes are uncertain.

Contingent consumption

Cambridge Dictionary

contingent /kənˈtɪn.dʒənt/ *adjective* (formal)

contingent on/upon something

depending on something else in the future in order to happen

States of nature

We refer to different outcomes of some random event as being different **states of nature**.

In this lecture, we are going to think about consumption under different states of nature as different goods.

For example, suppose my wealth is £100. A lottery ticket costs £5 and pays £200 if its number is drawn, and nothing otherwise.

	Wealth after the draw	
	Winning number drawn	**Not drawn**
Do not buy the ticket	£100	£100
Buy the ticket	$100 - 5 + 200 = £295$	$100 - 5 = £95$

Contingent consumption plan

A **contingent consumption plan** specifies consumption in every possible state. We choose the whole plan before learning which state occurs.

In the lottery example, I can choose between two plans:

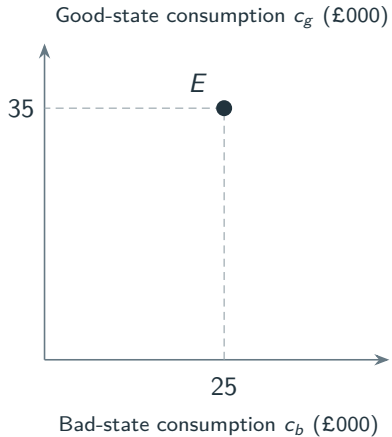
- **Do not buy the ticket**: my consumption will be £100 regardless of the lottery outcome.
- **Buy the ticket**: my consumption will be £295 if I win, and £95 if I lose.

Insurance and financial assets let us trade consumption across states.

Insurance as contingent consumption

Let's consider insurance in the framework of contingent consumption.

Initial wealth is £35,000. A loss of £10,000 occurs with probability $\pi = .01$.



Insurance as contingent consumption

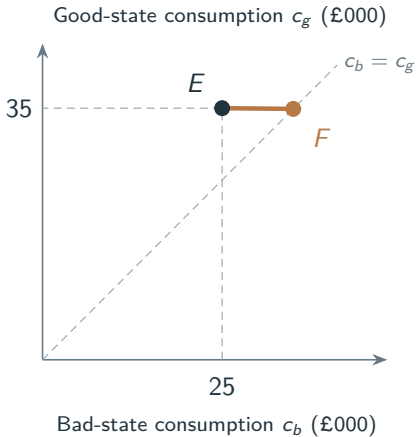
Let's consider insurance in the framework of contingent consumption.

Initial wealth is £35,000. A loss of £10,000 occurs with probability $\pi = .01$.

Suppose the consumer can buy insurance such that if he pays £1 in insurance premium, he is paid £100 in the bad state and nothing in the good state.

If the consumer pays £100, his consumption will be £34,900 in either state:

$$c_b = c_g = £34{,}900.$$



The insurance contract

In general, an insurance contract lets you pay a premium of γK, where K is the payout in the bad state. Let m be initial wealth and L the potential loss.

We can then write consumption under the contract:

$$c_b = m - L + K - \gamma K = m - L + (1 - \gamma)K,$$
$$c_g = m - \gamma K.$$

$K = 0$ corresponds to the case where the consumer buys no insurance.

$K = L$ corresponds to full insurance, $c_b = c_g$.

The contingent-consumption budget

Consumption in either state is given by

$$c_b = m - L + (1 - \gamma)K,$$
$$c_g = m - \gamma K.$$

In order to plot the budget line, we simply want an expression of c_g in terms of c_b:

$$K = \frac{c_b - m + L}{1 - \gamma},$$

$$c_g = m - \gamma \frac{c_b - m + L}{1 - \gamma}$$

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$$c_g = m - \gamma \frac{c_b - m + L}{1-\gamma}$$

$$= \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} c_b.$$

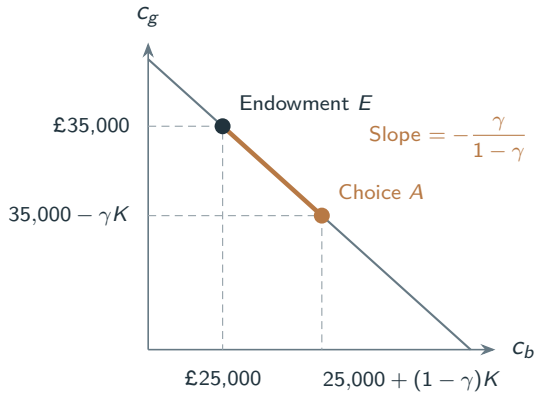
The insurance budget in a diagram

Buying K of cover costs γK and moves the consumer from E to A:

$$c_b = 25{,}000 + (1-\gamma)K,$$

$$c_g = 35{,}000 - \gamma K.$$

Each extra pound of consumption in the bad state costs $\gamma/(1-\gamma)$ pounds in the good state.



Utility and probabilities

Probabilities

If preferences about consumption in different states are "reasonable", we can still use utility functions to represent them.

Whether a consumer wants to transfer consumption across states depends on how likely each state is.

Let c_1, c_2 be consumption in two states with probabilities $\pi_1, \pi_2 > 0$, where $\pi_1 + \pi_2 = 1$. Preferences depend on both probabilities and consumption:

$$U(c_1, c_2; \pi_1, \pi_2).$$

Example: I am more eager to transfer resources to the "rainy state" if it is likely to rain.

Expected utility

An **expected utility function** or **von Neumann–Morgenstern utility function** is a probability-weighted sum of utility in each state:

$$U(c_1, c_2; \pi_1, \pi_2) = \pi_1 u(c_1) + \pi_2 u(c_2) = E[u(c)].$$

A **positive affine transformation** of U has the form $V = aU + b$, where $a > 0$ and b is any constant.

If this transformation is applied to an expected utility function, both the ranking of consumption plans and the expected-utility form are preserved.

Is expected utility reasonable?

Varian motivates expected utility through an **independence assumption**: the value of consumption in the state that occurs should not depend on consumption in states that do not occur. If I win the lottery, what I would have consumed if I had lost should not matter.

The expected utility form captures this idea. With three states:

$$U = \pi_1 u(c_1) + \pi_2 u(c_2) + \pi_3 u(c_3).$$

The marginal rate of substitution is then

$$MRS_{12} = -\frac{\partial U/\partial c_1}{\partial U/\partial c_2} = -\frac{\pi_1 \, \partial u(c_1)/\partial c_1}{\pi_2 \, \partial u(c_2)/\partial c_2}.$$

My willingness to trade consumption between states 1 and 2 does not depend on consumption in state 3.

Risk aversion

Expected wealth and expected utility

Suppose a consumer has £10. He can accept a gamble that gives a 50% chance of gaining £5 and a 50% chance of losing £5.

We must distinguish three objects:

$$E[W] = \tfrac{1}{2}(15) + \tfrac{1}{2}(5) = 10 \quad \text{expected wealth,}$$

$$E[u(W)] = \tfrac{1}{2}u(15) + \tfrac{1}{2}u(5) \quad \text{expected utility,}$$

$$u(E[W]) = u(10) \quad \text{utility of certain £10.}$$

The gamble and certain £10 have the same expected wealth, but their utilities need not be equal.

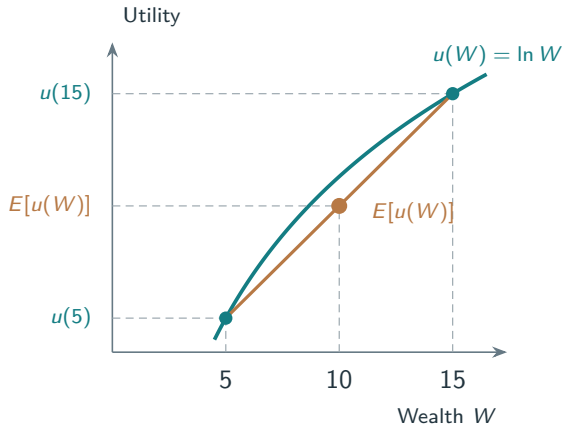
Risk aversion

If the utility function is strictly concave, expected utility, $E[u(W)]$, is smaller than the utility of expected wealth, $u(E[W])$.

At expected wealth £10:

$$u(10) > \tfrac{1}{2}u(5) + \tfrac{1}{2}u(15).$$

The consumer strictly prefers certain £10 to an uncertain outcome with expected wealth £10: the consumer is **risk averse**.



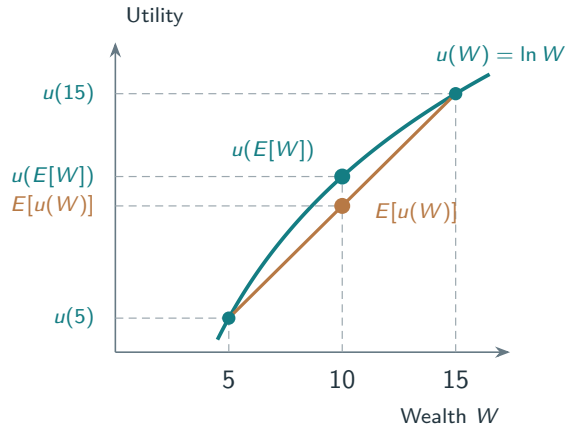
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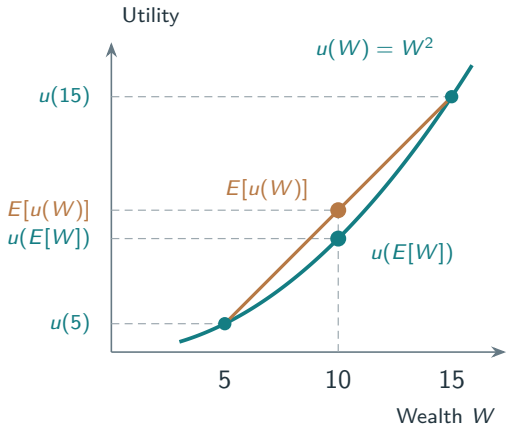


Risk loving

For a strictly convex utility function, expected utility exceeds the utility of expected wealth.

$$\tfrac{1}{2}u(5) + \tfrac{1}{2}u(15) > u(10).$$

The consumer prefers the gamble to a certain £10: the consumer is **risk loving**.

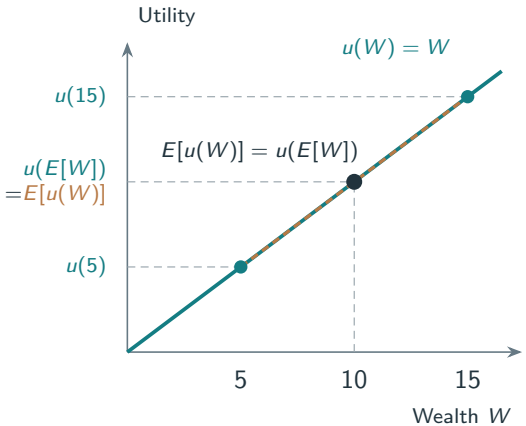


Risk neutrality

With a linear utility function u, expected utility and the utility of expected wealth coincide.

$$\tfrac{1}{2}u(5) + \tfrac{1}{2}u(15) = u(10).$$

The consumer is indifferent between the gamble and certain wealth: the consumer is **risk neutral**.



Insurance choice

Choosing insurance cover

Recall that under insurance

$$c_b = m - L + (1-\gamma)K,$$
$$c_g = m - \gamma K.$$

Let π be the probability of a loss. A **risk-averse consumer** chooses insurance cover K to maximise expected utility:

$$\max_{0 \le K \le L} EU(K) = \pi u(c_b) + (1-\pi)u(c_g).$$

or

$$\max_{0 \le K \le L} EU(K) = \pi u(\underbrace{m - L + (1-\gamma)K}_{c_b}) + (1-\pi)u(\underbrace{m - \gamma K}_{c_g}).$$

More cover (higher K) raises consumption in the loss state and lowers it in the no-loss state.

The marginal value of insurance

Differentiate the expected utility expression from the previous slide with respect to insurance cover K:

$$EU(K) = \pi u(m - L + (1-\gamma)K) + (1-\pi)u(m - \gamma K).$$

The chain rule gives

$$\begin{aligned}\frac{\partial EU}{\partial K} &= \pi \frac{\partial u(c_b)}{\partial c_b}\frac{\partial c_b}{\partial K} + (1-\pi)\frac{\partial u(c_g)}{\partial c_g}\frac{\partial c_g}{\partial K} \\ &= \pi(1-\gamma)\frac{\partial u(c_b)}{\partial c_b} - (1-\pi)\gamma\frac{\partial u(c_g)}{\partial c_g}.\end{aligned}$$

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$$= \pi(1-\gamma)\frac{\partial u(c_b)}{\partial c_b} - (1-\pi)\gamma\frac{\partial u(c_g)}{\partial c_g}.$$

The first term is the expected marginal benefit from higher consumption in the loss state. The second is the expected marginal cost from lower consumption in the no-loss state.

The optimal insurance choice

At an interior optimum ($0 < K < L$), the marginal change in expected utility is zero:

$$\frac{\partial EU}{\partial K} = \pi(1-\gamma)\frac{\partial u(c_b)}{\partial c_b} - (1-\pi)\gamma\frac{\partial u(c_g)}{\partial c_g} = 0$$

Rearranging gives

$$\pi(1-\gamma)\frac{\partial u(c_b)}{\partial c_b} = (1-\pi)\gamma\frac{\partial u(c_g)}{\partial c_g},$$

$$-\frac{\pi\,\partial u(c_b)/\partial c_b}{(1-\pi)\,\partial u(c_g)/\partial c_g} = -\frac{\gamma}{1-\gamma}.$$

The left side is the slope of the indifference curve. The right side is the slope of the budget line.

Actuarially fair insurance

Now let's consider the insurance contract from the insurance company's perspective.

With probability π, the insurer pays out K; otherwise, it pays nothing. In either state, it receives γK. Its expected profit is:

$$E[P] = \gamma K - [\pi K + (1 - \pi)0] = (\gamma - \pi)K.$$

Insurance is **actuarially fair** when the premium equals the expected payout. For $K > 0$:

$$(\gamma - \pi)K = 0 \quad \iff \quad \gamma = \pi.$$

Fair insurance and full cover

With actuarially fair insurance ($\gamma = \pi$), the optimality condition simplifies from

$$\pi(1-\gamma)\frac{\partial u(c_b)}{\partial c_b} = (1-\pi)\gamma\frac{\partial u(c_g)}{\partial c_g}.$$

to

$$\frac{\partial u(c_b)}{\partial c_b} = \frac{\partial u(c_g)}{\partial c_g}.$$

For a risk-averse consumer, this means equal consumption in both states.

$$\begin{aligned} m - L + (1-\pi)K &= m - \pi K \\ -L + K = 0 \quad &\Longrightarrow \quad K^* = L. \end{aligned}$$

A risk-averse consumer chooses full insurance when it is actuarially fair.

Diversification and risk sharing

Financial markets allow consumers to manage risk through **diversification** and **risk spreading**.

- **Diversification**: invest in assets whose returns do not move perfectly together and may offset one another. For example, sunglasses and raincoat companies perform differently depending on the weather.
- **Risk spreading**: pooling many independent individual risks reduces the risk borne by each member of the group. For example, health insurance spreads individuals' medical costs across a group.

When a shock affects many people or assets together, pooling and diversification cannot generally eliminate it. For example, an epidemic can affect many health insurance policyholders at once.