

LECTURE 5

Intertemporal Choice

5SSPP217 – Intermediate Microeconomics

Recap: market demand

Market demand for a good is the total quantity demanded by all consumers at a given set of prices and incomes:

$$X^1(p, m) = \sum_{i=1}^n x_i^1(p_1, p_2, m_i).$$

An own-price change induces a **movement along** the market demand curve. Changes in incomes or the prices of other goods **shift** the curve.

Inverse demand, $p = P(X)$, gives the price at which consumers demand a certain quantity X .

Recap: price elasticity and revenue

Price elasticity is a unit-free measure of demand's response to price:

$$\varepsilon = \frac{\partial X / X}{\partial p / p} = \frac{p}{X} \frac{\partial X}{\partial p}, \quad R(p) = pX(p).$$

At $\varepsilon = -2$, a 1% price rise reduces demand by about 2%.

$$\frac{\partial R(p)}{\partial p} = X(p) + p \frac{\partial X(p)}{\partial p} = X(p)(1 + \varepsilon).$$

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For downward-sloping demand:

Elasticity	Demand	A small price increase
$ \varepsilon > 1$	Elastic	Reduces revenue
$ \varepsilon = 1$	Unit elastic	Does not affect revenue (to first order)
$ \varepsilon < 1$	Inelastic	Increases revenue

Recap: marginal revenue

Marginal revenue measures the additional revenue from a small increase in quantity:

$$R(X) = P(X)X, \quad MR = \frac{\partial R(X)}{\partial X} = P(X) + X \frac{\partial P(X)}{\partial X}.$$

Marginal revenue is less than price because selling additional units requires a lower price, reducing revenue on all existing sales.

For downward-sloping demand:

$$MR = P(X) \left(1 - \frac{1}{|\epsilon|} \right) < P(X).$$

Recap: income elasticity

Income elasticity measures the responsiveness of demand to income:

$$\eta_i = \frac{\partial x_i / x_i}{\partial m_i / m_i} = \frac{m_i}{x_i} \frac{\partial x_i}{\partial m_i}.$$

- **Inferior:** $\eta_i < 0$; demand falls as income rises.
- **Normal:** $\eta_i > 0$; demand rises as income rises.
- **Luxury:** $\eta_i > 1$; demand rises proportionately more than income.
- **Necessity:** $0 < \eta_i < 1$; demand rises proportionately less than income.

Intertemporal Choice

Consumption over time

Intertemporal choice is the choice of how much to consume at different points in time.

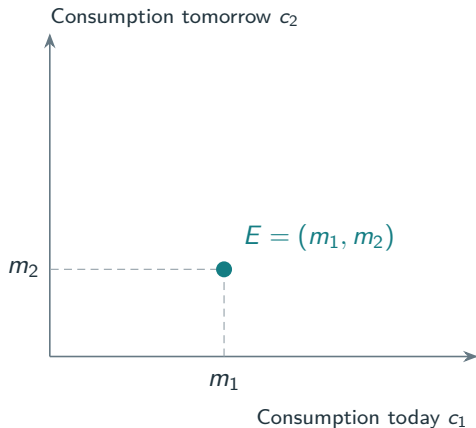
Consider a consumer who has to choose how much to consume in each of two time periods, (c_1, c_2) , given his income in each period, (m_1, m_2) , his **endowment**.

Consumption without saving or borrowing

Suppose the consumer can neither save nor borrow.

The consumer can at most consume m_1 in period 1 and m_2 in period 2, and because “more is better” (monotonicity), the endowment point is the optimal consumption bundle:

$$(c_1^*, c_2^*) = (m_1, m_2) = E.$$

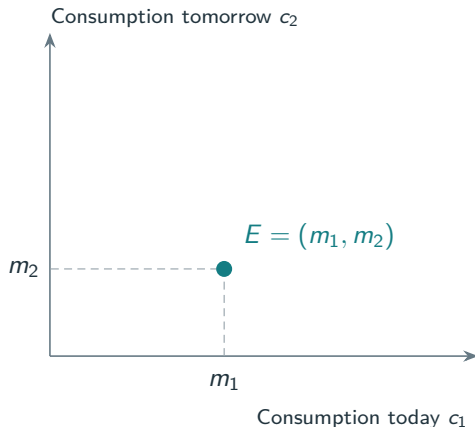


Consumption **with** saving and **without** borrowing

Suppose now that the consumer can save his period 1 income at interest rate r , but still cannot borrow.

Maximum consumption in period 2 equals income m_2 plus savings $(s = m_1 - c_1)$ and interest $(r \cdot s)$ from period 1:

$$c_2 = m_2 + s + rs$$



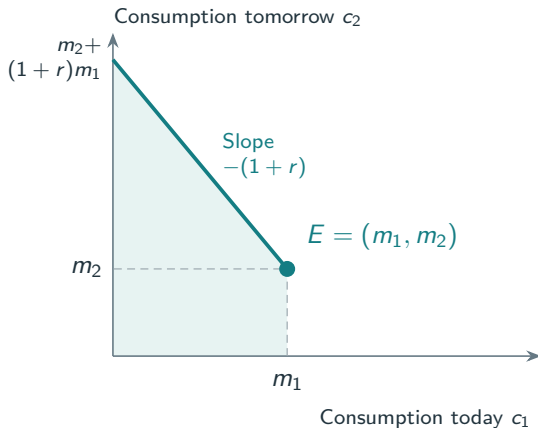
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$$\begin{aligned}c_2 &= m_2 + s + rs \\&= m_2 + (1 + r)s \\&= m_2 + (1 + r)(m_1 - c_1).\end{aligned}$$

Giving up one unit of consumption today buys $1 + r$ units tomorrow.

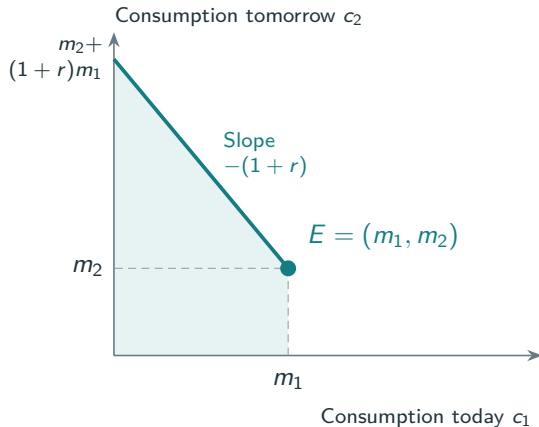


Consumption **with** saving and **with** borrowing

Suppose now that the consumer can also borrow at the same interest rate r .

If $c_1 > m_1$, borrowing $b = c_1 - m_1 > 0$ must be repaid with interest tomorrow:

$$\begin{aligned}c_2 &= m_2 - (1+r)b \\ &= m_2 + (1+r)(m_1 - c_1).\end{aligned}$$

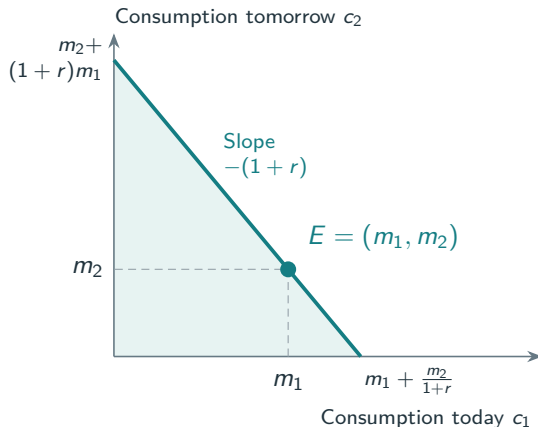


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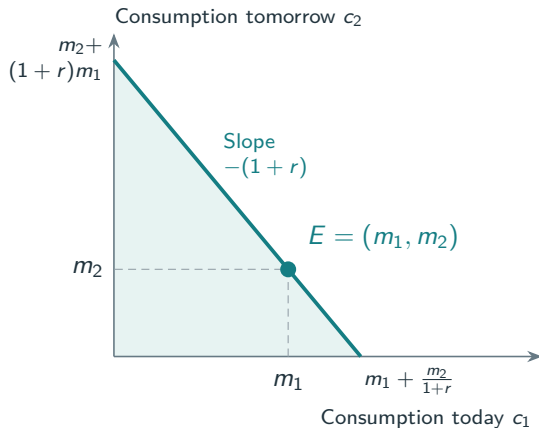
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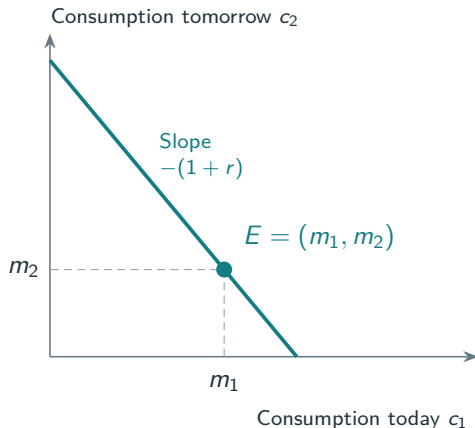
$$\begin{aligned}c_1 &= \frac{(1+r)m_1 + m_2 - c_2}{1+r} \\ &= m_1 + \frac{m_2 - c_2}{1+r}.\end{aligned}$$



A change in the interest rate

Suppose the interest rate falls from r to $r' < r$.

- The endowment is affordable whatever the interest rate.



A change in the interest rate

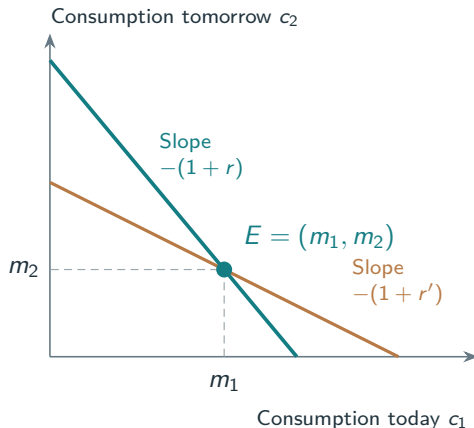
Suppose the interest rate falls from r to $r' < r$.

- The endowment is affordable whatever the interest rate.
- Maximum consumption in period 2 falls:

$$c_2^{\max} = m_2 + (1 + r')m_1.$$

- Maximum consumption in period 1 rises:

$$c_1^{\max} = m_1 + \frac{m_2}{1 + r'}.$$



Present-value and future-value budgets

We can rearrange the budget constraint:

$$c_2 = m_2 + (1 + r)(m_1 - c_1)$$

$$(1 + r)c_1 + c_2 = (1 + r)m_1 + m_2.$$

Since everything is expressed in terms of period-2 values, this is referred to as the **future-value** budget constraint.

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We can divide both sides by $1 + r$ to obtain the **present-value** budget constraint, where all terms are expressed in period-1 values:

$$c_1 + \frac{c_2}{1 + r} = m_1 + \frac{m_2}{1 + r}.$$

Preferences for consumption

Preferences across time

As before, we can use indifference curves to represent preferences for different bundles of period 1 and period 2 consumption, $u(c_1, c_2)$.

- **Perfect substitutes:** $u = c_1 + c_2$. The consumer values a unit today and a unit tomorrow equally.
- **Perfect complements:** $u = \min\{c_1, c_2\}$. The consumer wants equal quantities in both periods.
- **Strictly convex preferences:** e.g., $u = c_1^a c_2^b$, with $a, b > 0$. The optimal consumption bundle will likely feature some c_1 and some c_2 , depending on the interest rate, r .

Optimal consumption

The consumer chooses the most-preferred affordable plan:

$$\max_{c_1, c_2 \geq 0} u(c_1, c_2) \quad \text{subject to} \quad (1+r)c_1 + c_2 \leq (1+r)m_1 + m_2.$$

If there is a smooth, interior optimum, then:

$$MRS = -\frac{\partial u / \partial c_1}{\partial u / \partial c_2} = -(1+r).$$

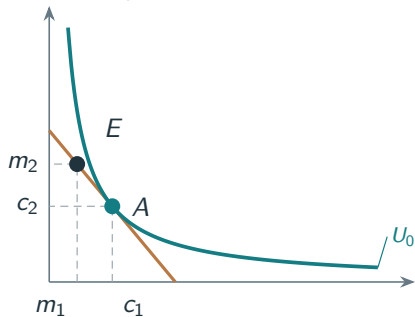
The consumer trades off consumption in periods 1 and 2 at a price $1+r$.

Comparative statics

Borrowers and lenders

Borrower: $c_1 > m_1$, $c_2 < m_2$.

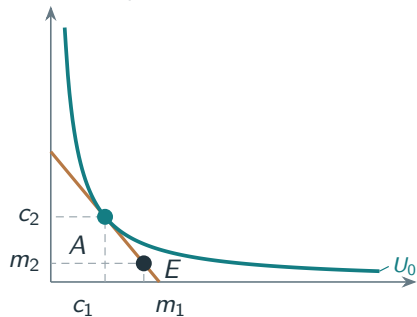
Consumption tomorrow c_2



Consumption today c_1

Lender: $c_1 < m_1$, $c_2 > m_2$.

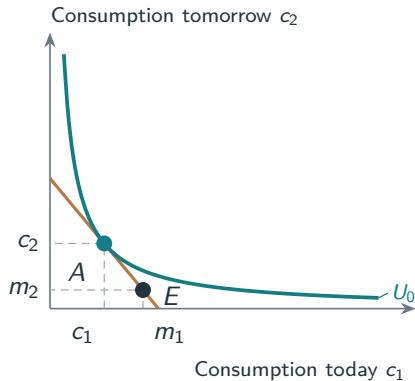
Consumption tomorrow c_2



Consumption today c_1

Lenders: an increase in r

The consumer has an endowment of $E = (m_1, m_2)$ and optimally chooses bundle $A = (c_1, c_2)$, where $c_1 < m_1$.

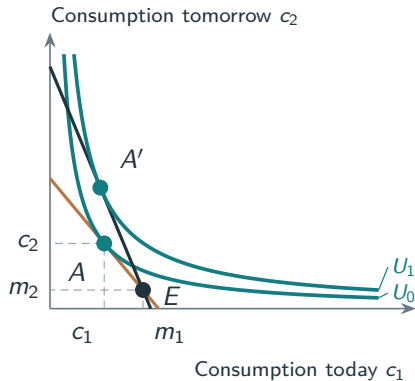


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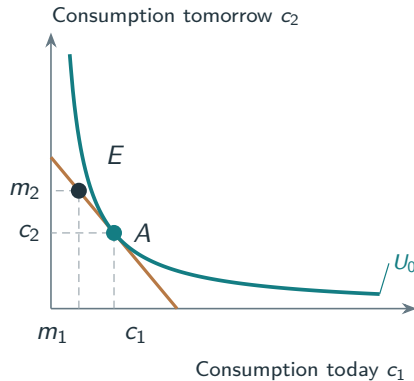
When r rises, the budget line pivots around the endowment and A' becomes the new optimal consumption bundle.

The consumer remains a lender and is now better off.



Borrowers: an increase in r

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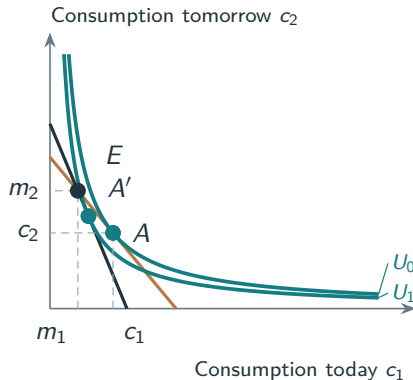


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When r rises, the budget line pivots around the endowment and A' becomes the new optimal consumption bundle.

The consumer may remain a borrower or become a lender.



Interest-rate changes and welfare

Initial position	Rate change	Conclusion
Lender	Rise	Remains a lender, better off.
Borrower	Fall	Remains a borrower, better off.
Borrower	Rise	Worse off if still a borrower.
Lender	Fall	Worse off if still a lender.

The Slutsky equation and intertemporal choice

The price of current consumption

As in last week's lecture, the **Slutsky equation** decomposes the effect of a price change into substitution and income effects.

Let's define $q = 1 + r$ as the relative price of current consumption, and $W = qm_1 + m_2$ as the future value of the endowment. Hence $\Delta q = \Delta r$.

An increase in r raises the relative price q of consuming today, and also raises the future value W of the endowment.

Recall: the Slutsky decomposition

Recall the Slutsky decomposition: the total change in consumption is the sum of the substitution effect and the income effect.

$$\underbrace{\Delta c_1^*}_{\text{total change}} = \underbrace{\Delta c_1^s}_{\text{substitution effect}} + \underbrace{\Delta c_1^m}_{\text{income effect}} .$$

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- **Substitution effect:** change the price from q to q' and adjust income so that the original bundle remains affordable.
- **Income effect:** hold the new price q' fixed and change income from the compensated level W^s to the actual level W' .

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$$\underbrace{\Delta c_1^*}_{\text{total change}} = \underbrace{\Delta c_1^s}_{\text{substitution effect}} + \underbrace{\Delta c_1^m}_{\text{income effect}} .$$

$$\begin{aligned} \Delta c_1^* = & \underbrace{c_1(q', W^s) - c_1(q, W)}_{\text{substitution effect}} \\ & + \underbrace{c_1(q', W') - c_1(q', W^s)}_{\text{income effect}} . \end{aligned}$$

Compensated and actual income

Suppose the interest rate increases from r to r' , so q increases to $q' = q + \Delta q$. The income needed to consume the original optimal bundle (c_1^*, c_2^*) at the new price is

$$\begin{aligned} W^s &= q' c_1^* + c_2^* \\ &= q c_1^* + c_1^* \Delta q + c_2^* \\ &= W + c_1^* \Delta q. \end{aligned}$$

The actual new income is $W' = W + m_1 \Delta q$, so

$$\begin{aligned} \Delta W^m &= W' - W^s \\ &= W + m_1 \Delta q - W - c_1^* \Delta q \\ &= (m_1 - c_1^*) \Delta q. \end{aligned}$$

The intertemporal Slutsky equation

Dividing the Slutsky equation by Δq , we get

$$\frac{\Delta c_1^*}{\Delta q} = \frac{\Delta c_1^s}{\Delta q} + \frac{\Delta c_1^m}{\Delta q}.$$

We can write the income effect as

$$\begin{aligned}\Delta c_1^m &= \frac{\Delta c_1^m}{\Delta W^m} \Delta W^m \\ &= \underbrace{\frac{\Delta c_1^m}{\Delta W^m}}_{\text{consumption response to income}} \underbrace{(m_1 - c_1^*) \Delta q}_{\text{income change}}.\end{aligned}$$

For normal current consumption, substituting Δc_1^m gives

$$\frac{\Delta c_1^*}{\Delta q} = \frac{\Delta c_1^s}{\Delta q} + (m_1 - c_1^*) \frac{\Delta c_1^m}{\Delta W^m}.$$

(?) (-) (?) (+)

Why borrowers and lenders respond differently

If current consumption is a **normal good**, the income response $\Delta c_1^m / \Delta W^m$ is positive. For an increase in r :

	Substitution	Net income	Effect on c_1
Borrower	Nonpositive	Negative	Falls
Lender	Nonpositive	Positive	Ambiguous

For borrowers, the income effect reduces c_1 and substitution is nonpositive.

For lenders, the income effect increases c_1 and substitution is nonpositive.

Inflation

Inflation and the consumption budget

So far, we have assumed that the price of consumption is the same in periods 1 and 2.

Now let $p_1 = 1$ and $p_2 = 1 + \pi$. Hold the endowments m_1, m_2 fixed in units of consumption.

The **nominal** rate r measures the return in money. The **real** rate ρ measures the return in units of consumption.

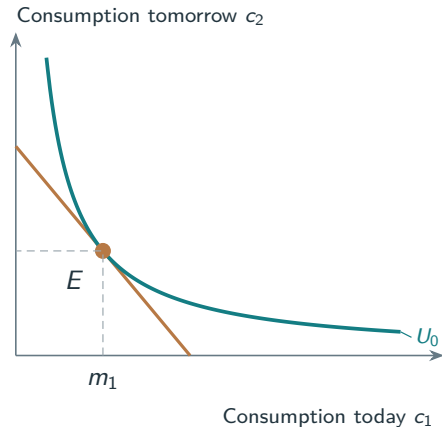
$$1 + \rho = \frac{1 + r}{1 + \pi}, \quad c_2 = m_2 + \frac{1 + r}{1 + \pi}(m_1 - c_1) \\ = m_2 + (1 + \rho)(m_1 - c_1).$$

Present value: a closer look

Higher present value expands consumption possibilities

If the consumer can borrow and lend safely at the same interest rate r , then a higher present-value endowment corresponds to a parallel budget line farther out.

In this example, the consumer prefers E to E' if they must consume the endowment directly. With borrowing and saving, E' allows them to reach a higher indifference curve.



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