

LECTURE 4

Market demand

Intermediate Microeconomics

Recap: individual demand

Individual demand: $x_1^* = x_1(p_1, p_2, m)$.

Income changes (p_1, p_2 fixed)

Normal good: $\frac{\partial x_1}{\partial m} > 0$, Inferior good: $\frac{\partial x_1}{\partial m} < 0$.

Own-price changes (p_2, m fixed)

Ordinary good: $\frac{\partial x_1}{\partial p_1} < 0$, Giffen good: $\frac{\partial x_1}{\partial p_1} > 0$.

Other-price changes (p_1, m fixed)

Substitutes: $\frac{\partial x_1}{\partial p_2} > 0$, Complements: $\frac{\partial x_1}{\partial p_2} < 0$.

Recap: curves and inverse demand

Income offer curve: the set of optimal consumption bundles as income, m changes.

Engel curve: the relationship between income and quantity demanded of one good.

Price offer curve: the set of optimal consumption bundles as p_1 changes.

Inverse demand function: the price that makes a given quantity optimal:

$$p_1 = p_1(x_1, p_2, m).$$

Recap: the Slutsky equation

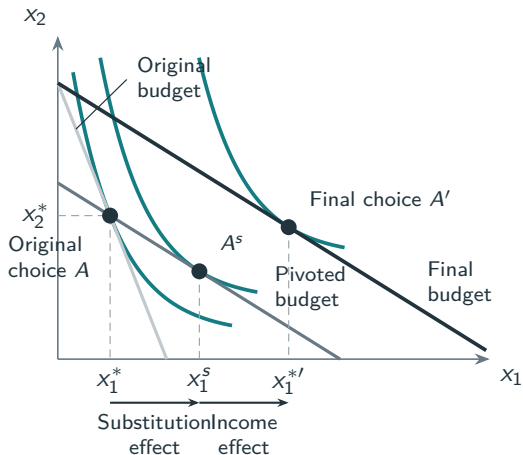
The behavioural response to a price change can be decomposed into:

Substitution effect ($A \rightarrow A^s$)

$$\Delta x_1^s = x_1(p'_1, p_2, m^s) - x_1(p_1, p_2, m).$$

Income effect ($A^s \rightarrow A'$)

$$\Delta x_1^m = x_1(p'_1, p_2, m) - x_1(p'_1, p_2, m^s).$$



Market demand

From individual to market
demand

Market demand

So far, we have looked at individual demand. We will now look at how we can construct market demand.

There are n consumers. Consumer i has income m_i and demand for good 1:

$$x_{1i} = x_{1i}(p_1, p_2, m_i).$$

Market demand is the sum of individual demands:

$$X_1(p_1, p_2, m_1, \dots, m_n) = \sum_{i=1}^n x_{1i}(p_1, p_2, m_i).$$

The **market demand curve** denotes how market demand for a good depends on its own price, given prices of other goods and individual incomes:

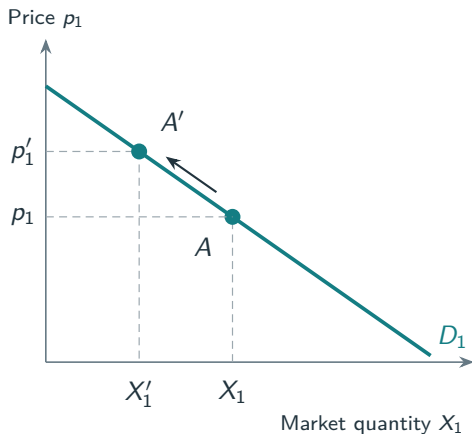
$$X_1 = D_1(p_1, p_2, m_1, \dots, m_n) = D_1(p, m)$$

Movements along and shifts of the market demand curve

Market demand is given by

$$D_1(p_1, p_2, m).$$

An increase in the price of good 1 to p'_1 gives a **movement along** its demand curve from A where $X_1 = D_1(p_1, p_2, m)$ to A' where $X_1 = D_1(p'_1, p_2, m)$.



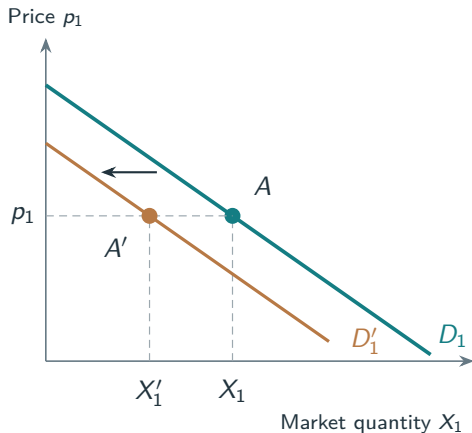
Movements along and shifts of the market demand curve

Market demand is given by

$$D_1(p_1, p_2, m).$$

If good 2 is a complement, an increase in p_2 to p'_2 **shifts** the demand curve inward.

For the same p_1 , quantity falls from X_1 to X'_1 .



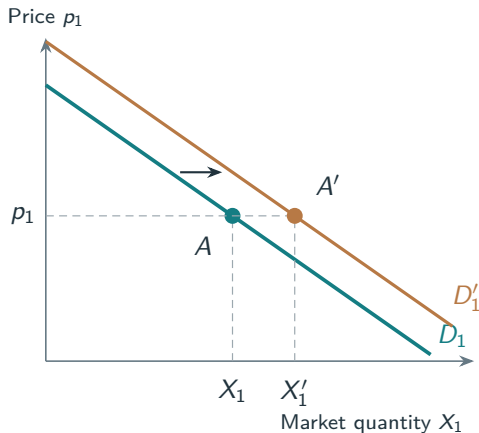
Movements along and shifts of the market demand curve

Market demand is given by

$$D_1(p_1, p_2, m).$$

If good 2 is a substitute, an increase in p_2 to p'_2 **shifts** the demand curve outward.

For the same p_1 , quantity rises from X_1 to X'_1 .



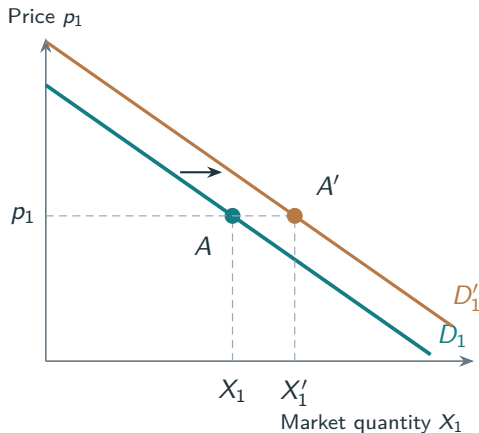
Movements along and shifts of the market demand curve

Market demand is given by

$$D_1(p_1, p_2, m).$$

An increase in income to m' **shifts** the demand curve outward for a normal good.

For the same p_1 , quantity rises from X_1 to X'_1 .



Income distribution and market demand

Suppose that consumers A and B have the **same quasilinear utility function**:

$$u_i(x_{1i}, x_{2i}) = 10 \ln x_{1i} + x_{2i}, \quad p_1 = p_2 = 1, \quad m_A = 5, m_B = 15$$

With these utility functions, each consumer wants at most 10 units of good 1 and spends any remaining income on good 2:

$$x_{1i} = \min\{10, m_i\}, \quad x_{2i} = m_i - x_{1i} \geq 0.$$

With $(m_A, m_B) = (5, 15)$,

$$(x_{1,A}, x_{2,A}) = (5, 0), \quad (x_{1,B}, x_{2,B}) = (10, 5), \quad (X_1, X_2) = (15, 5)$$

If we transfer 5 units of income from B to A , i.e., $m_A, m_B = (10, 10)$:

$$(x_{1,A}, x_{2,A}) = (10, 0), \quad (x_{1,B}, x_{2,B}) = (10, 0) \quad (X_1, X_2) = (20, 0)$$

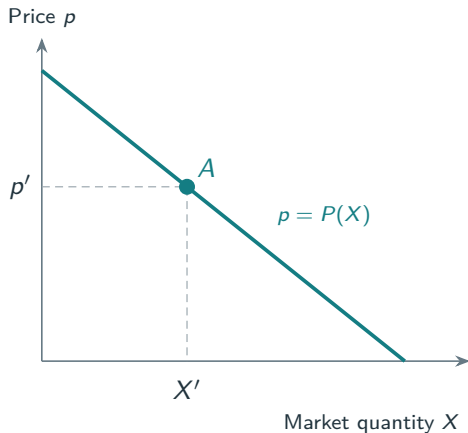
The inverse demand function

Ordinary and inverse market demand

Instead of asking the question: what is the quantity demanded at a certain price, we can ask: what is the price that induces individuals to demand a certain quantity?

The latter perspective is referred to as **inverse demand**:

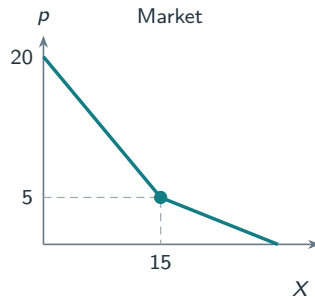
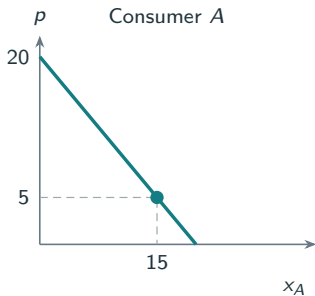
$$p = P(X).$$



Adding individual demand curves

Horizontal addition: at each price, the quantities demanded are added.

$$D_A(p) = \max\{20 - p, 0\}, \quad D_B(p) = \max\{10 - 2p, 0\}.$$



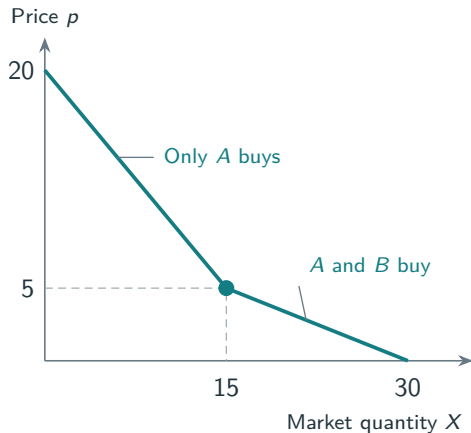
Kinked market demand

Ordinary market demand is given by

$$D(p) = \begin{cases} 0, & p \geq 20, \\ 20 - p, & 5 < p < 20, \\ 30 - 3p, & 0 \leq p \leq 5. \end{cases}$$

Inverse demand is given by

$$P(X) = \begin{cases} 20 - X, & 0 < X \leq 15, \\ 10 - X/3, & 15 < X \leq 30. \end{cases}$$



Discrete goods

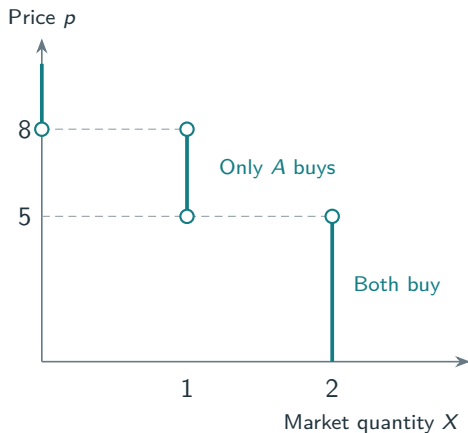
Reservation prices and discrete demand

Each consumer buys either zero or one unit. Their **reservation price** r_i is the price at which they are indifferent between buying and not buying.

$$x_i(p) = \begin{cases} 1, & p < r_i, \\ 0, & p > r_i. \end{cases}$$

Example: $r_A = 8$ and $r_B = 5$.

Market demand counts the consumers whose reservation prices exceed p .



The extensive and the intensive margin

Changes in market demand reflect changes along the intensive and extensive margin.

Intensive margin: consumers who already buy the good change how much they buy.

Extensive margin: consumers enter or leave the market.

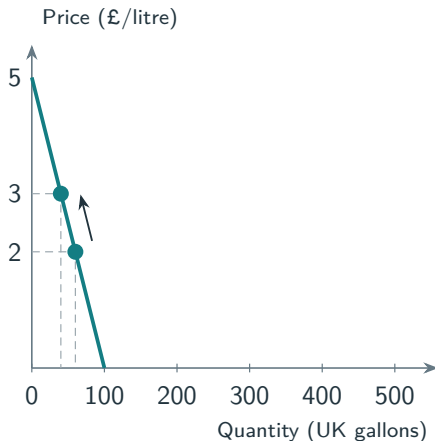
Elasticity

The responsiveness of demand to prices

If we want to measure the responsiveness of demand to prices, it seems intuitive to simply take the slope of the demand curve.

Here, gasoline is measured in UK (imperial) gallons. The slope, $\Delta q / \Delta p$, is $-100/5 = -20$.

100 UK gallons $\approx$ 454.6 litres.

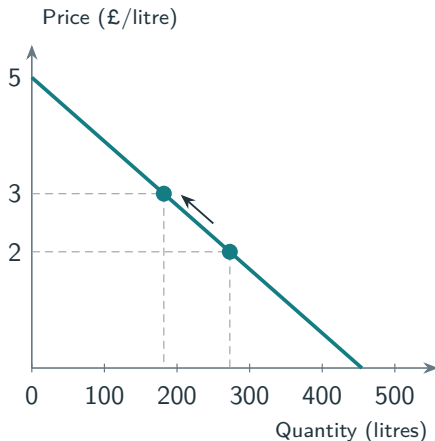


The responsiveness of demand to prices

If we want to measure the responsiveness of demand to prices, it seems intuitive to simply take the slope of the demand curve.

Here, gasoline is measured in litres. The slope, $\Delta q / \Delta p$, is $-454.6/5 \approx -90.9$.

100 UK gallons ≈ 454.6 litres.



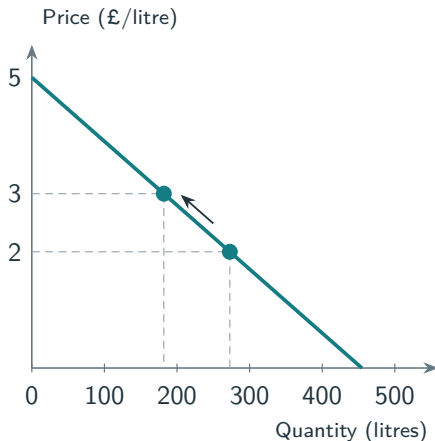
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100 UK gallons $\approx$ 454.6 litres.

The slope of the demand curve depends on the unit used to measure quantity.



Price elasticity of demand

The **price elasticity** of demand is a unit-free measure of responsiveness. It measures the percentage response of quantity demanded to a percentage change in price:

$$\varepsilon(p) = \frac{\frac{\partial X}{\partial p}}{\frac{X}{p}} = \frac{p}{\partial p} \frac{\partial X(p)}{X(p)} = \frac{p}{X(p)} \frac{\partial X(p)}{\partial p}.$$

Example: If $\varepsilon = -2$, a 1% price increase reduces demand by 2%.

Elasticity of a linear demand curve

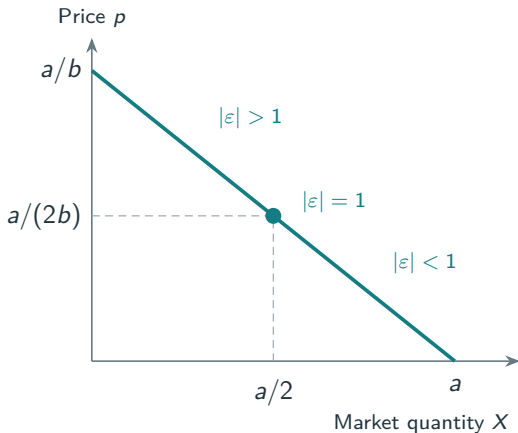
For $X(p) = a - bp$,

$$\frac{\partial X}{\partial p} = -b, \quad \varepsilon(p) = -\frac{bp}{a - bp}.$$

The slope of the demand curve is constant, but the elasticity depends on p .

At the midpoint,

$$p = \frac{a}{2b}, \quad X = \frac{a}{2}, \quad \varepsilon = -1.$$



Elastic and inelastic demand

Because elasticities of demand are usually negative, we tend to use its absolute value

Elastic demand: $|\varepsilon| > 1$. A 1% price increase reduces quantity by more than 1%.

Unit elastic demand: $|\varepsilon| = 1$. Quantity changes by the same percentage as price.

Inelastic demand: $|\varepsilon| < 1$. A 1% price increase reduces quantity by less than 1%.

If two goods are close substitutes, demand tends to be more elastic as buyers can easily switch to the other good when the price rises.

Elasticity and revenue

Revenue and a price change

Revenue is defined as price times quantity:

$$R(p) = pX(p).$$

If we increase p , revenue becomes:

$$\begin{aligned} R' &= (p + \Delta p)(q + \Delta q) \\ &= pq + q\Delta p + p\Delta q + \Delta p \Delta q. \end{aligned}$$

For small changes in price, $\Delta p \Delta q \approx 0$:

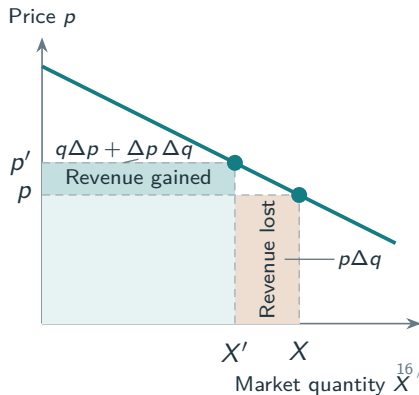
$$\Delta R = q\Delta p + p\Delta q$$

$$\frac{\Delta R}{\Delta p} = q + p \frac{\Delta q}{\Delta p}$$

When does an increase in p increase R ?

$$\frac{\Delta R}{\Delta p} > 0 \iff q + p \frac{\Delta q}{\Delta p} > 0$$

$$\begin{aligned} p \frac{\Delta q}{\Delta p} > -q &\iff \frac{p}{q} \frac{\Delta q}{\Delta p} > -1 \\ &\Rightarrow |\varepsilon(p)| < 1 \end{aligned}$$



Elasticity and the change in revenue

In general, the effect of price on revenue can be obtained by taking the derivative of $R(p)$ with respect to p :

$$\begin{aligned}\frac{\partial R}{\partial p} &= X + p \frac{\partial X}{\partial p} \\ &= X \left(1 + \frac{p}{X} \frac{\partial X}{\partial p} \right) = X(1 + \varepsilon).\end{aligned}$$

For downward-sloping demand,

$$\frac{\partial R}{\partial p} = X(1 - |\varepsilon|).$$

- **Inelastic:** a small price increase raises revenue.
- **Elastic:** a small price increase lowers revenue.
- **Unit elastic:** revenue is unchanged to first order.

Constant elasticity demands

Constant elasticity demand

A **constant elasticity** demand function has the form

$$X(p) = Ap^{\varepsilon}.$$

Its price elasticity is the same at every price:

$$\frac{p}{X} \frac{\partial X}{\partial p} = \frac{p}{Ap^{\varepsilon}} \left(\varepsilon Ap^{\varepsilon-1} \right) = \varepsilon.$$

Unit elastic demand

When $\varepsilon = -1$,

$$X(p) = Ap^{-1}, \quad R(p) = pX(p) = A.$$

Revenue is constant along the whole demand curve.

Example: $X = 12/p$.

$$p = 6 \implies X = 2, \quad R = 12,$$

$$p = 2 \implies X = 6, \quad R = 12.$$



Elasticity and marginal revenue

Marginal revenue

We can write revenue as a function of the quantity sold:

$$R(X) = P(X)X.$$

Marginal revenue is the additional revenue from a small increase in quantity:

$$MR(X) = \frac{\partial R}{\partial X} = P(X) + XP'(X).$$

- $P(X)$ is the revenue from an additional unit sold.
- $XP'(X)$ is the change in revenue on the units already sold when the price changes.

For downward-sloping inverse demand, $P'(X) < 0$, so $MR(X) < P(X)$ for $X > 0$: the additional revenue from selling another unit is less than the price of that unit.

Marginal revenue and elasticity

For ordinary goods, where $D'(p) < 0$:

$$\varepsilon = \frac{p}{X} \frac{\partial X}{\partial p} = \frac{p}{X} D'(p), \quad P'(X) = \frac{1}{D'(p)}$$

$$\Rightarrow XP'(X) = \frac{X}{D'(p)} = \frac{p}{\varepsilon} = \frac{P(X)}{\varepsilon}.$$

Substituting into $MR = P + XP'$ gives

$$MR = P(X) \left(1 + \frac{1}{\varepsilon} \right) = P(X) \left(1 - \frac{1}{|\varepsilon|} \right).$$

- **Elastic demand:** $MR > 0$; more output raises revenue.
- **Unit elastic demand:** $MR = 0$.
- **Inelastic demand:** $MR < 0$; more output lowers revenue.

Marginal revenue curves

Marginal revenue with linear demand

For a linear inverse demand function,

$$P(X) = a - bX$$

revenue and marginal revenue are

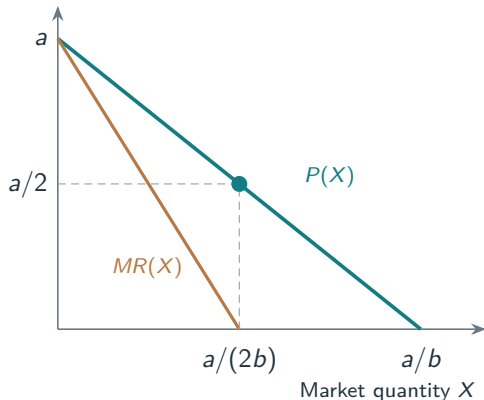
$$R(X) = aX - bX^2,$$

$$MR(X) = a - 2bX.$$

At $X = 0$, $P(0) = MR(0) = a$, but the slope of the marginal revenue curve is twice that of the inverse demand curve.

Intuition: To sell another unit, a firm must lower the price, but this reduces revenue from the goods already being sold.

Price / marginal revenue



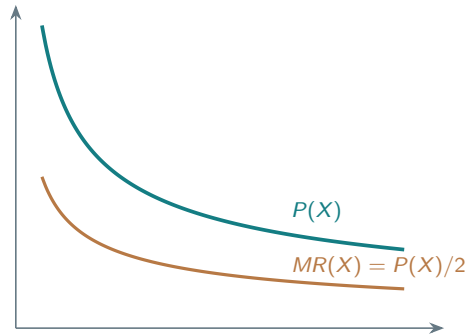
Marginal revenue with constant elasticity

For the constant elasticity demand,
 $X = Ap^{-2}$, marginal revenue is

$$MR(X) = P(X) \left(1 - \frac{1}{|\varepsilon|} \right) = P(X)/2.$$

- If $|\varepsilon| > 1$, MR is a positive fraction of price.
- If $|\varepsilon| = 1$, $MR = 0$ at every quantity.
- If $|\varepsilon| < 1$, $MR < 0$ at every quantity.

Price / marginal revenue



Market quantity X

Income elasticity

Income elasticity of demand

The **income elasticity of demand** measures how demand responds to a change in income:

$$\eta_i = \frac{\frac{\partial x_i}{x_i}}{\frac{\partial m}{m}} = \frac{m}{x_i} \frac{\partial x_i}{\partial m}.$$

- **Inferior good:** $\eta_i < 0$.
- **Normal good:** $\eta_i > 0$.
- **Luxury:** $\eta_i > 1$; quantity rises proportionately more than income.
- **Necessity:** $0 < \eta_i < 1$; quantity rises proportionately less than income.

If the entire budget is spent, the weighted average of the income elasticities equals 1.