

LECTURE 3

Demand and the Slutsky equation

Intermediate Microeconomics

Recap: utility

A **utility function** assigns a number to every consumption bundle such that more-preferred bundles receive larger numbers than less-preferred bundles:

$$(x_1, x_2) \succ (y_1, y_2) \iff u(x_1, x_2) > u(y_1, y_2).$$

Marginal utility is change in utility as consumption of one good increases, holding consumption of the other good fixed:

$$MU_1 = \frac{\partial u(x_1, x_2)}{\partial x_1}, \quad MU_2 = \frac{\partial u(x_1, x_2)}{\partial x_2}.$$

The **marginal rate of substitution** measures how much of good 2 the consumer is willing to give up for an additional unit of good 1, while keeping utility constant:

$$MRS = \frac{dx_2}{dx_1} = -\frac{MU_1}{MU_2}.$$

Recap: choice

Consumers choose the most-preferred bundle from their budget sets.

If there is an interior optimum, use the **Lagrangian method** to find it:

$$\max_{x_1, x_2} \mathcal{L} = u(x_1, x_2) + \lambda(m - p_1x_1 - p_2x_2).$$

Demand functions give the optimal quantities of each good as functions of prices and income:

$$x_1^* = x_1(p_1, p_2, m),$$

$$x_2^* = x_2(p_1, p_2, m).$$



Demand

Comparative statics

Comparing two equilibria – and ignoring the transition from one to the other – is known as **comparative statics**.

In this lecture, we will study how demand for a good changes when prices or incomes change.

Normal and inferior goods

Normal goods

How does demand for a good change when income m increases to m' ?

$$x_1^* = x_1(p_1, p_2, m)$$

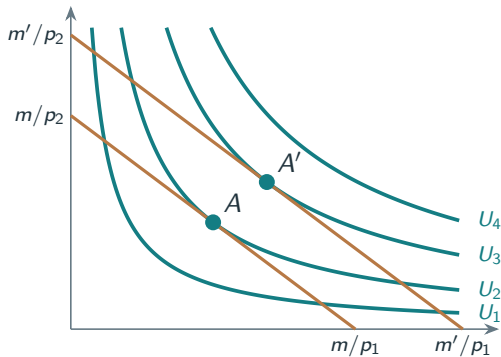
vs.

$$x_1^{*'} = x_1(p_1, p_2, m')$$

If the consumption of a good increases when m increases, we call that good a **normal good**:

$$\frac{\Delta x_1}{\Delta m} > 0.$$

Quantity of good 2



Quantity of good 1

Inferior goods

If the consumption of a good decreases when m increases, we call that good an **inferior good**:

$$\frac{\Delta x_1}{\Delta m} < 0.$$

Example

Normal good: restaurant food

Inferior good: canned soup



Income offer curves and Engel curves

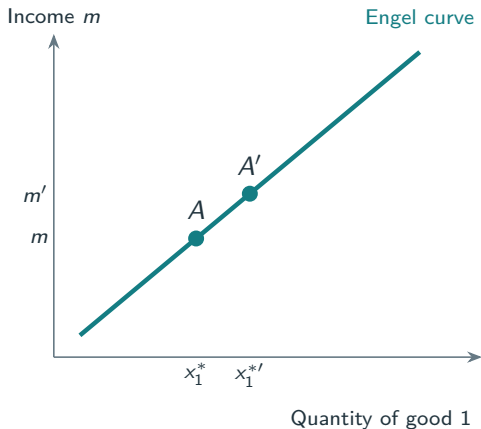
Income offer curves

The **income offer curve** shows the set of optimal bundles for different levels of income, m .



Engel curves

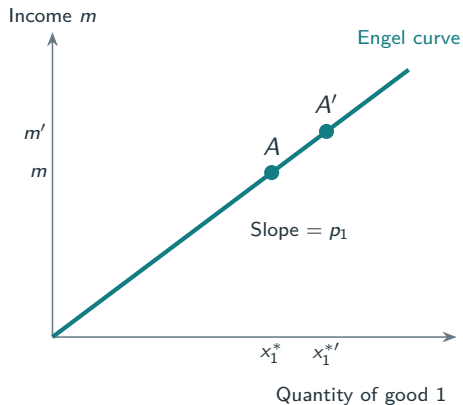
An **Engel curve** shows the relationship between income and the quantity demanded of one good, holding prices fixed.



Some examples

Perfect substitutes: $p_1 < p_2$

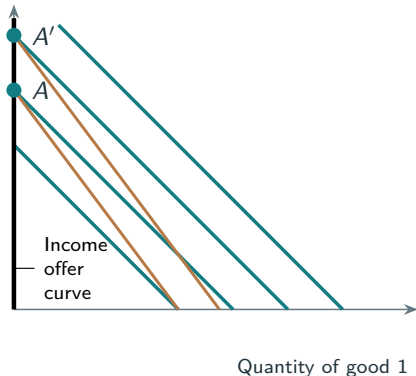
$$u(x_1, x_2) = x_1 + x_2. \quad p_1 < p_2: x_1^* = m/p_1 \text{ and } x_2^* = 0.$$



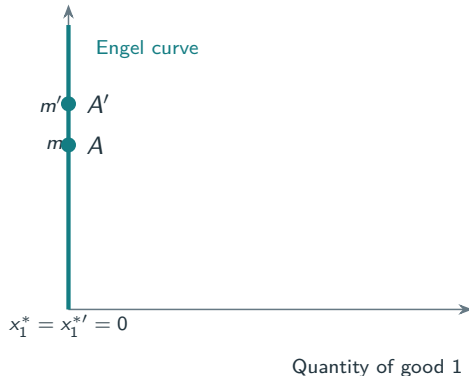
Perfect substitutes: $p_1 > p_2$

$$u(x_1, x_2) = x_1 + x_2. \quad p_1 > p_2: x_1^* = 0 \text{ and } x_2^* = m/p_2.$$

Quantity of good 2



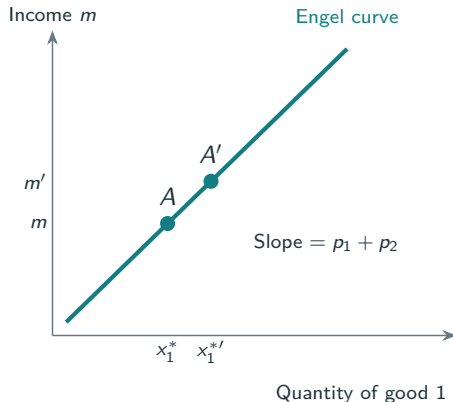
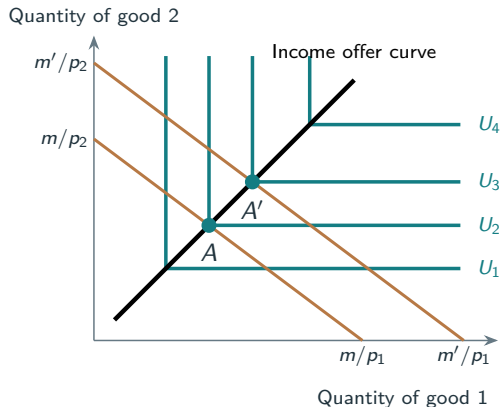
Income m



Takeaway: all additional income will be spent on the good that is cheapest.

Perfect complements

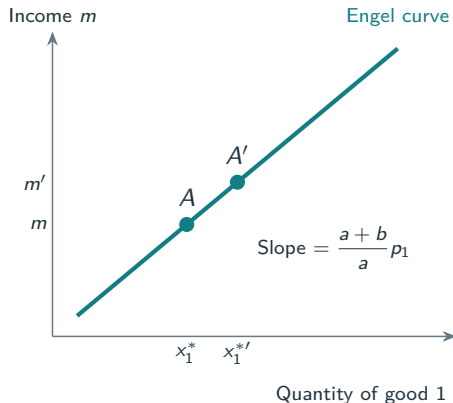
$$u(x_1, x_2) = \min\{x_1, x_2\}. \quad x_1^* = x_2^* = \frac{m}{p_1 + p_2}.$$



Takeaway: additional income will be spent on the goods in fixed proportions.

Cobb–Douglas preferences

$$u(x_1, x_2) = x_1^a x_2^b. \quad x_1^* = \frac{a}{a+b} \frac{m}{p_1} \text{ and } x_2^* = \frac{b}{a+b} \frac{m}{p_2}.$$



Takeaway: Demand for each good increases proportionally with income.

Homothetic preferences

Preferences are **homothetic** if multiplying two bundles by the same positive scalar preserves their preference ranking:

$$\mathbf{x} \succeq \mathbf{y} \iff t\mathbf{x} \succeq t\mathbf{y} \quad \text{for every } t > 0.$$

Examples: perfect substitutes, perfect complements, and Cobb–Douglas preferences.

Implications:

- Scaling income by a factor t scales optimal consumption of each good by the same factor.
- Income offer curves and Engel curves are straight lines through the origin.

Quasilinear preferences

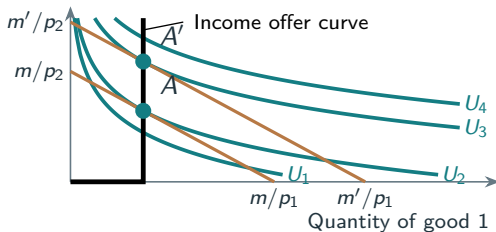
$u(x_1, x_2) = v(x_1) + x_2$, with
 $v(x_1) = \ln x_1$.

$$x_1^* = \begin{cases} \frac{m}{p_2}, & m \leq p_2, \\ \frac{p_1}{p_2}, & m > p_2. \end{cases}$$

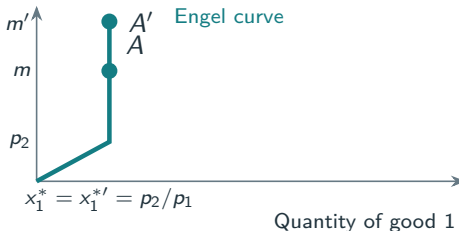
$$x_2^* = \max\left\{0, \frac{m}{p_2} - 1\right\}.$$

Takeaway: when x_1 reaches beyond a certain level, all additional income is spent on good 2.

Quantity of good 2



Income m



Ordinary goods and Giffen goods

Ordinary goods

How does demand for a good change when income p_1 increases to p'_1 ?

$$x_1^* = x_1(p_1, p_2, m)$$

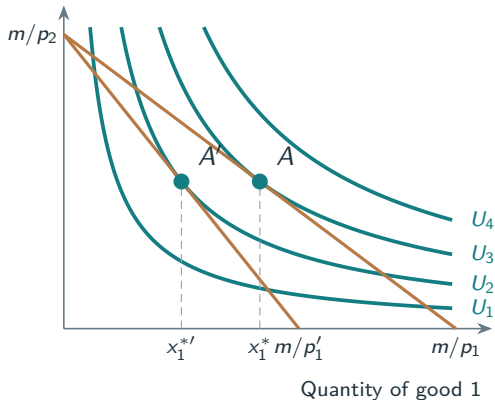
vs.

$$x_1^{*'} = x_1(p'_1, p_2, m)$$

If the consumption of a good decreases when p_1 increases, we call that good an **ordinary good**:

$$\frac{\Delta x_1}{\Delta p_1} < 0.$$

Quantity of good 2

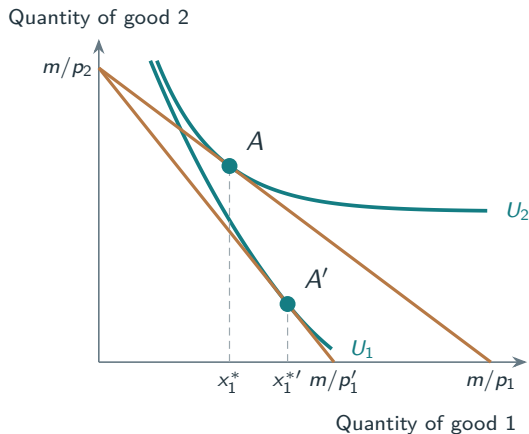


Giffen goods

If the consumption of a good increases when p_1 increases, we call that good an **Giffen good**:

$$\frac{\Delta x_1}{\Delta p_1} > 0.$$

A Giffen good must be inferior, but an inferior good need not be Giffen.



Sir Robert Giffen (1837–1910)

Alfred Marshall's *Principles of Economics* (3rd ed., 1895) credited Giffen with the idea that demand curves may not always slope downwards.

There is actually no record of it in any of Giffen's writings (Stigler, 1947)

Do Giffen goods exist in reality?

- Samuelson (1964) popularised the idea that potatoes during the **Irish potato famine** were a Giffen good.
- Jensen and Miller (2008) found evidence that rice is a Giffen good for poor households in Hunan, China.

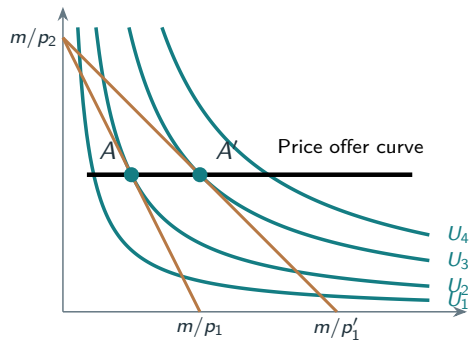


The price offer curve and the demand curve

The price offer curve and the demand curve

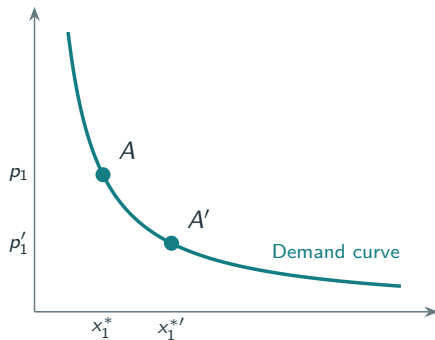
The price offer curve shows the set of optimal bundles for different levels of the price of good 1, p_1

Quantity of good 2



Quantity of good 1

Price p_1



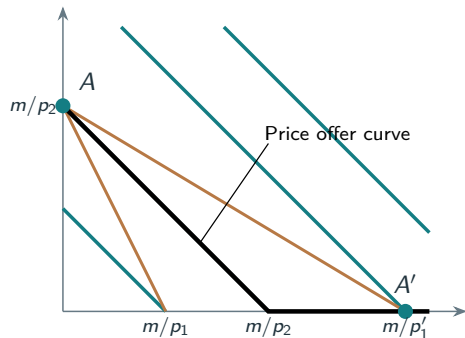
Quantity of good 1

Some examples

Perfect substitutes

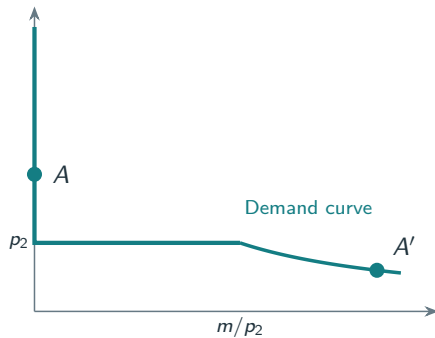
$$u(x_1, x_2) = x_1 + x_2. \quad x_1^* = m/p_1 \text{ if } p_1 < p_2; \quad x_1^* = 0 \text{ if } p_1 > p_2; \quad x_1^* \in [0, m/p_2] \text{ if } p_1 = p_2.$$

Quantity of good 2



Quantity of good 1

Price p_1

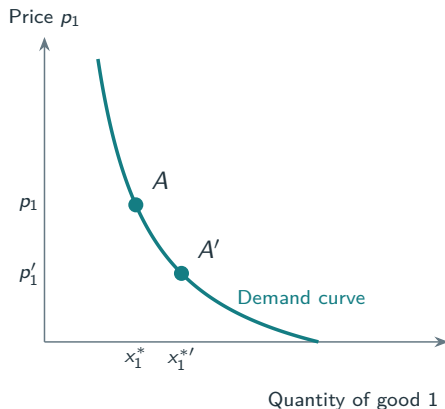
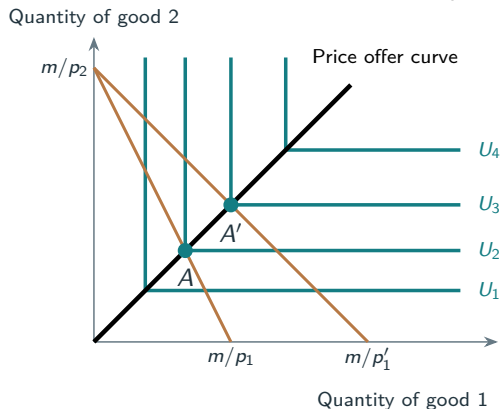


Quantity of good 1

Takeaway: as p_1 falls below p_2 , the consumer switches entirely to good 1.

Perfect complements

$$u(x_1, x_2) = \min\{x_1, x_2\}. \quad x_1^* = x_2^* = \frac{m}{p_1 + p_2}.$$



Takeaway: a fall in p_1 raises demand for both goods, while the one-to-one consumption ratio stays fixed.

Substitutes and complements

Substitutes and complements

How does x_1 change when p_2 changes, holding p_1 and m fixed?

Good 1 is a **substitute** for good 2 if a rise in p_2 increases demand for good 1:

$$\frac{\partial x_1(p_1, p_2, m)}{\partial p_2} > 0.$$

Good 1 is a **complement** to good 2 if a rise in p_2 reduces demand for good 1:

$$\frac{\partial x_1(p_1, p_2, m)}{\partial p_2} < 0.$$

The inverse demand function

The inverse demand function

The **inverse demand function**

measures what the price of good 1 would have to be for that level of consumption to be optimal:

$$p_1 = p_1(x_1, p_2, m).$$

Example: $u(x_1, x_2) = \ln x_1 + x_2$ and $p_2 = 1$, with both goods consumed:

$$x_1^* = \frac{1}{p_1} \iff p_1 = \frac{1}{x_1^*}.$$

At an interior optimum, price equals marginal willingness to pay:

$$p_1 = \frac{MU_1}{MU_2} = -MRS \quad (p_2 = 1).$$



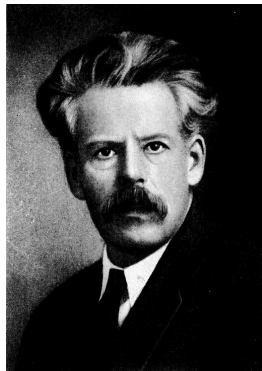
Slutsky equation

Evgeny “Eugenio” Slutsky

Evgeny Slutsky (1880–1948) was a Russian economist and statistician.

In 1915, he published his contribution to consumer theory (in Italian...), separating the response to a price change into two effects.

- **Substitution effect:** a change in relative prices changes the goods the consumer chooses.
- **Income effect:** a change in purchasing power changes what the consumer can afford.



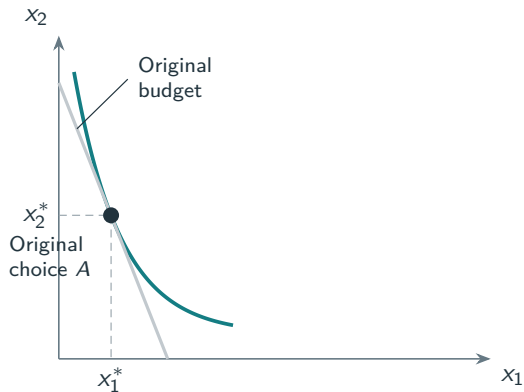
Evgeny Slutsky

The substitution effect

The substitution effect

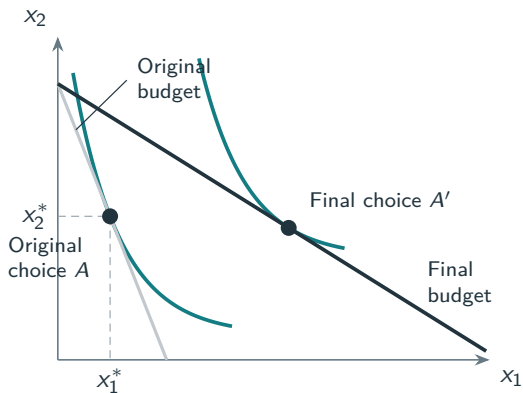
At p_1 , the optimal bundle is

$$A = (x_1^*, x_2^*).$$



The substitution effect

At p_1 , the optimal bundle is $A = (x_1^*, x_2^*)$. A fall to p'_1 changes the optimal bundle to A' .



The substitution effect

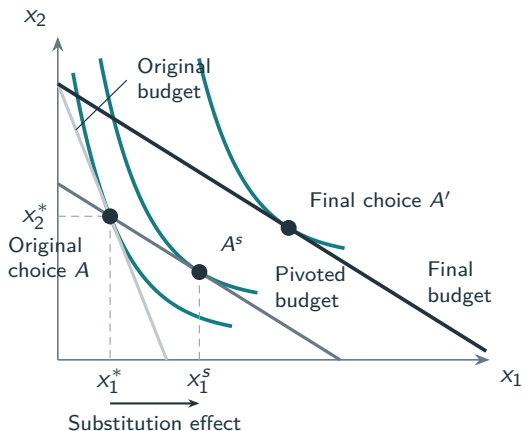
At p_1 , the optimal bundle is $A = (x_1^*, x_2^*)$. A fall to p'_1 changes the optimal bundle to A' .

The pivoted budget, m^s , is the income necessary to afford the original bundle at the new price:

$$m^s = m + (p'_1 - p_1)x_1^*.$$

The **Slutsky substitution effect** is equal to the change in demand on the original and pivoted budgets:

$$\Delta x_1^s = x_1(p'_1, p_2, m^s) - x_1(p_1, p_2, m).$$

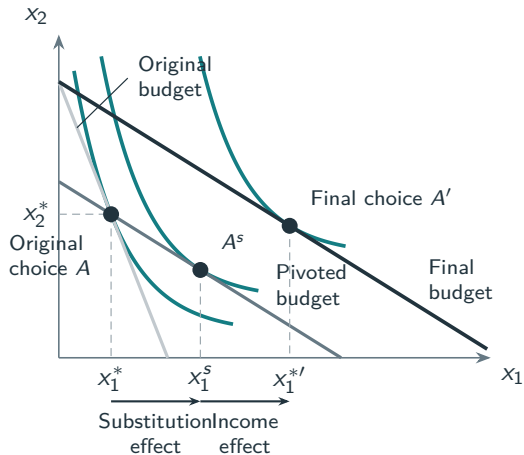


The income effect

The **income effect** is the change in demand on the pivoted and final budgets:

$$\Delta x_1^m = x_1(p'_1, p_2, m) - x_1(p'_1, p_2, m^s).$$

Note that the income effect can be positive (normal goods) or negative (inferior goods).



Sign of the substitution effect

Sign of the substitution effect

Contrary to the income effect, the substitution effect must always be zero or negative:

$$(p'_1 - p_1)(x_1^s - x_1^*) \leq 0.$$

- If p_1 rises, compensated demand, x_1^s , for good 1 cannot increase.

Why?

- Bundle A and A^s cost the same at the new price:

$$p'_1 x_1^* + p_2 x_2^* = p'_1 x_1^s + p_2 x_2^s.$$

- Bundle A^s is preferred to the original bundle, A , at the original price. By revealed preference, bundle A^s must therefore cost more than bundle A at the original price:

$$p_1 \Delta x_1 + p_2 \Delta x_2 \geq 0.$$

Takeaway: normal, inferior, and Giffen goods all obey the compensated law of demand.

The total change in demand

The total change in demand

The total price effect is the move from A to A' :

$$\Delta x_1 = x_1^{*'} - x_1^*.$$

Add and subtract the compensated choice x_1^s :

$$\begin{aligned}\Delta x_1 &= (x_1^s - x_1^*) \\ &\quad + (x_1^{*'} - x_1^s) \\ &= \Delta x_1^s + \Delta x_1^m.\end{aligned}$$

This is the **Slutsky decomposition**.

