

LECTURE 2

Utility and Choice

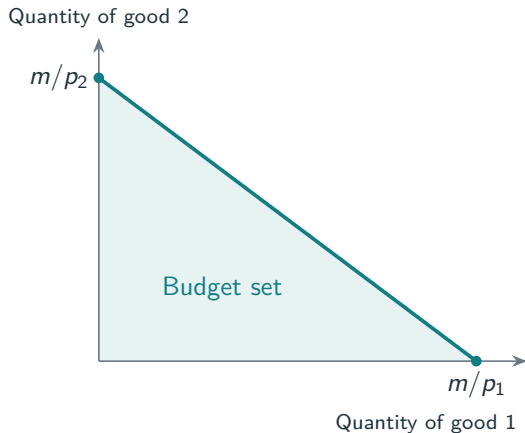
Intermediate Microeconomics

Recap: constraints and preferences

Recap: constraints

The **budget set** consists of all consumption bundles that are affordable at the given prices and income:

$$x_1 p_1 + x_2 p_2 \leq m.$$



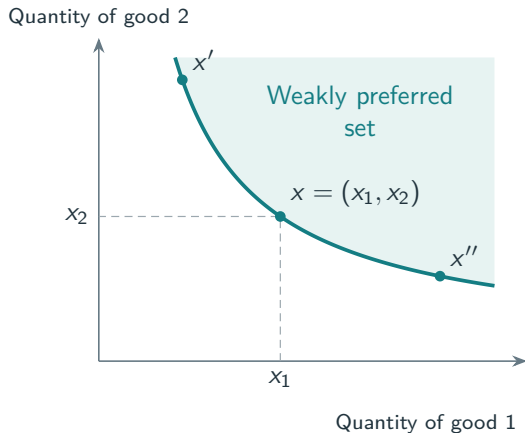
Recap: preferences

Axioms of consumer theory

- Completeness
- Reflexivity
- Transitivity

Well-behaved preferences

- Monotonicity
- Convexity



Utility

History of utility

The economic concept of **(marginal) utility** was developed independently by William Stanley Jevons, Carl Menger and Léon Walras in the early 1870s.

Jevons drew on Bentham's utilitarian philosophy, which posited that society ought to maximise utility or general happiness, but Menger and Walras did not.

Early “marginalists” treated utility as if it were measurable, but later economists moved to a more “**ordinal**” interpretation of utility: utility can represent the ranking of alternatives, but there is no meaning in its actual magnitude.

Under an ordinal interpretation, interpersonal comparisons of utility are not permitted.

For a critical appraisal of “utility”, see Joan Robinson's *Economic Philosophy*.

Utility function

A **utility function** is a way of assigning a number to every possible consumption bundle such that more-preferred bundles get assigned larger numbers than less-preferred bundles:

$$(x_1, x_2) \succ (y_1, y_2) \quad \text{if and only if} \quad u(x_1, x_2) > u(y_1, y_2).$$

Because only the ranking matters, if there is one way of assigning utilities to bundles, then there is an infinite number of ways.

Any **monotonic transformation** of a utility function is a utility function that represents the same preference ranking.

Constructing a utility function

Constructing a utility function

Can we always find a utility function to describe preferences?

Suppose a person prefers A over B, B over C, and C over A (intransitive preferences):

$$A \succ B \succ C \succ A \implies u(A) > u(B) > u(C) > u(A),$$

which is a contradiction.

But most “reasonable” preferences can be represented by a continuous utility function.

Some examples of utility functions

Perfect substitutes

The consumer is willing to trade good 1 for good 2 at a constant rate.

The utility function could be:

$$u(x_1, x_2) = x_1 + x_2.$$

or:

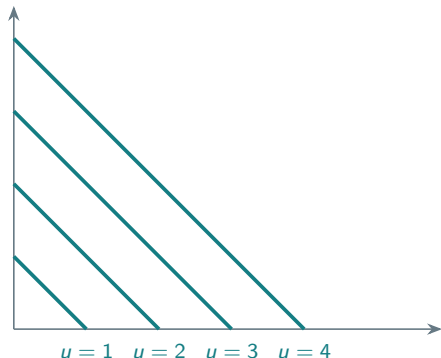
$$u(x_1, x_2) = 2x_1 + 2x_2.$$

Indifference curves are straight lines:

$$x_2 = k - x_1.$$

Example: red and blue pencils, when the consumer does not care about their colour.

Quantity of good 2



Quantity of good 1

Perfect complements

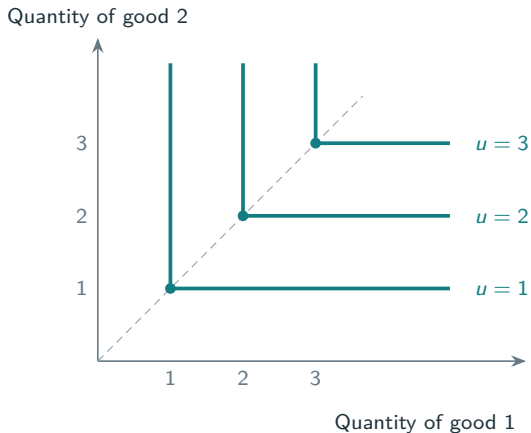
The consumer consumes good 1 and good 2 in fixed proportions.

The utility function could be:

$$u(x_1, x_2) = \min\{x_1, x_2\}.$$

Indifference curves are L-shaped, with corners at $x_1 = x_2$.

Example: left and right shoes.



Quasilinear utility

Utility is linear in one good and may be nonlinear in the other (hence “quasi”):

$$u(x_1, x_2) = v(x_1) + x_2.$$

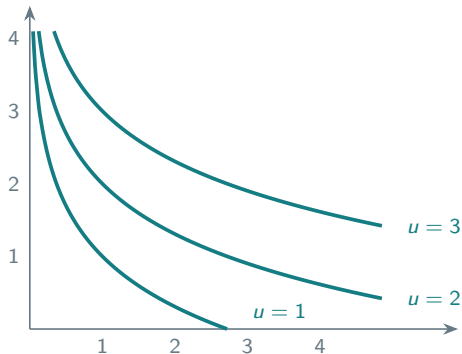
$$x_2 = k - v(x_1)$$

All indifference curves have the same slope at any given x_1 :

$$\frac{dx_2}{dx_1} = -v'(x_1).$$

Example: $u(x_1, x_2) = \ln x_1 + x_2$

Quantity of good 2



Quantity of good 1

Cobb–Douglas utility

A commonly used utility function is

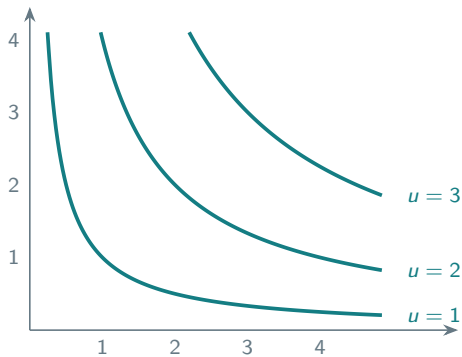
$$u(x_1, x_2) = x_1^a x_2^b, \quad a, b > 0.$$

Indifference curves are convex and slope downwards:

$$x_2 = \frac{k^{\frac{1}{b}}}{x_1^{\frac{a}{b}}}.$$

Example: $u(x_1, x_2) = x_1^{0.5} x_2^{0.5}$.

Quantity of good 2



Quantity of good 1

Marginal utility

Marginal utility

How does a consumer's utility change when we increase good 1 or good 2 by one?

This change in utility is known as **marginal utility**:

$$MU_1 = \frac{\Delta U}{\Delta x_1} = \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1}.$$

$$MU_2 = \frac{\Delta U}{\Delta x_2} = \frac{u(x_1, x_2 + \Delta x_2) - u(x_1, x_2)}{\Delta x_2}.$$

Note that when we change x_1 , we keep the quantity of x_2 fixed, and vice versa.

Marginal utility and the marginal rate of substitution

The marginal rate of substitution measures how much the consumer is willing to give up of good 2 for an additional unit of good 1.

Suppose we change the consumption of both goods in such a way to keep utility constant:

$$MU_1\Delta x_1 + MU_2\Delta x_2 = \Delta U = 0,$$

$$MRS = \frac{\Delta x_2}{\Delta x_1} = -\frac{MU_1}{MU_2}.$$

The magnitude of marginal utility is not interpretable, but the ratio of marginal utilities very cleanly maps onto the MRS.

Choice

Optimal choice

The most-preferred affordable bundle

Consumers choose the most-preferred bundle from their budget sets.

Mathematically, we say that they maximise utility by choosing x_1 and x_2 subject to their budget constraint:

$$\max_{x_1, x_2 \geq 0} u(x_1, x_2)$$

subject to $p_1x_1 + p_2x_2 \leq m$.

Graphically, the **optimal choice** is found at the highest indifference curve that intersects with the budget line (because of monotonicity).



An interior optimum

In this example, U_2 is **tangent** to the budget line at the optimal choice A:

$$MRS = -\frac{MU_1}{MU_2} = -\frac{p_1}{p_2}.$$



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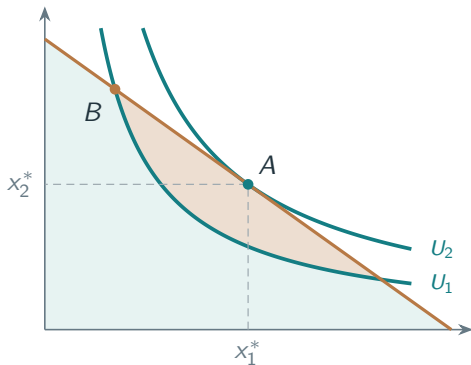
$$MRS = -\frac{MU_1}{MU_2} = -\frac{p_1}{p_2}.$$

At B , the indifference curve U_1 intersects the budget line without being tangent.

The **shaded area** contains all affordable bundles **strictly preferred to B** : point B cannot be optimal.

With **convex** preferences, any point that satisfies the tangency condition must be an optimal point.

Quantity of good 2

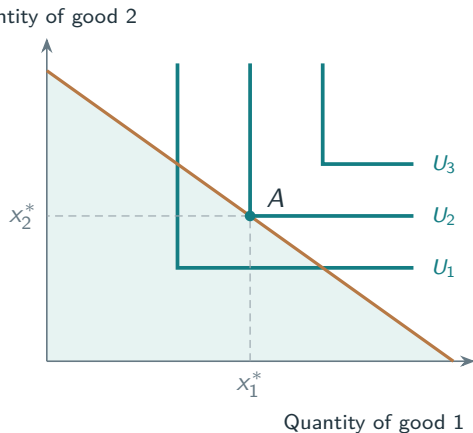


Quantity of good 1

Should the indifference curve always be tangent to the budget line at the optimal point?

The optimal choice is still reached at the highest affordable indifference curve, U_2 .

At the optimal choice A , the indifference curve has a kink and, mathematically, the tangent is not defined.



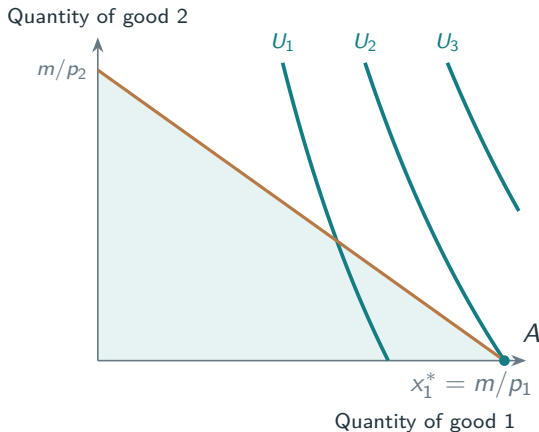
Corner solutions

Corner solutions are another example where the budget line need not be tangent to the indifference curve at the optimal point.

The consumer chooses

$$x_1^* = \frac{m}{p_1}, \quad x_2^* = 0.$$

At the market prices, the consumer would like to give up more of good 2 for good 1, but consumption of good 2 cannot fall below zero.



Finding an interior optimum: the Lagrangian

If the utility function is differentiable, we can use the Lagrangian method to find the interior optimum:

$$\mathcal{L} = u(x_1, x_2) + \lambda(m - p_1x_1 - p_2x_2).$$

To find the optimum, differentiate $\mathcal{L}$ with respect to x_1 , x_2 , and λ . This gives the following first-order conditions:

$$MU_1 - \lambda p_1 = 0,$$

$$MU_2 - \lambda p_2 = 0,$$

$$m - p_1x_1 - p_2x_2 = 0.$$

Dividing the first two conditions gives $MU_1/MU_2 = p_1/p_2$.

At the optimum, λ is the **marginal utility of income**: how much utility would rise if income rose by one unit.

Example: quasilinear utility

Suppose that utility is quasilinear: $u(x_1, x_2) = \ln x_1 + x_2$.

The Lagrangian is:

$$\mathcal{L} = \ln x_1 + x_2 + \lambda(m - p_1 x_1 - p_2 x_2).$$

The first-order conditions are:

$$\frac{1}{x_1} = \lambda p_1, \quad 1 = \lambda p_2, \quad p_1 x_1 + p_2 x_2 = m.$$

The second condition gives $\lambda = 1/p_2$. Substituting into the first condition and then the budget constraint gives:

$$x_1^* = \frac{p_2}{p_1}, \quad x_2^* = \frac{m}{p_2} - 1.$$

Consumer demand

Consumer demand

The optimal quantities of good 1 and good 2 consumption depend on prices and income:

$$x_1^* = x_1(p_1, p_2, m),$$

$$x_2^* = x_2(p_1, p_2, m).$$

These are the consumer's **demand functions**.

To find demand, solve the maximisation problem for any prices and income.

Some examples

Consumer demand: perfect substitutes

Preferences are represented by:

$$u(x_1, x_2) = x_1 + x_2$$

Consider three cases: $p_1 < p_2$,

$p_1 = p_2$, $p_1 > p_2$

$$x_1^* = \begin{cases} m/p_1, & p_1 < p_2, \\ [0, m/p_1], & p_1 = p_2, \\ 0, & p_1 > p_2. \end{cases}$$



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Consumer demand: perfect complements

Preferences are represented by

$$u(x_1, x_2) = \min\{x_1, x_2\}$$

At the optimum, consumption of good 1 is equal to that of good 2 and the budget is exhausted:

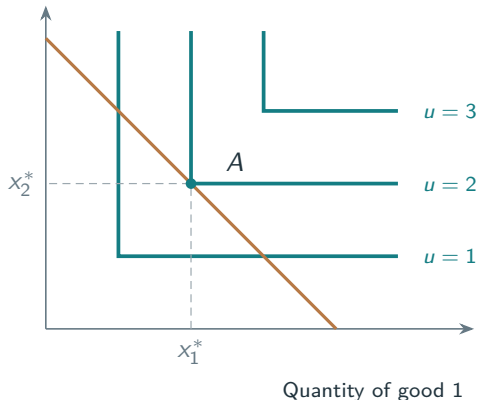
$$x_1^* = x_2^*,$$

$$p_1 x_1^* + p_2 x_2^* = m.$$

Therefore,

$$x_1^* = x_2^* = \frac{m}{p_1 + p_2}.$$

Quantity of good 2



Consumer demand: Cobb–Douglas

Preferences are represented by

$$u(x_1, x_2) = x_1^a x_2^b$$

$$\frac{MU_1}{MU_2} = \frac{ax_1^{a-1}x_2^b}{bx_1^ax_2^{b-1}} = \frac{ax_2}{bx_1} = \frac{p_1}{p_2}.$$

Combining this condition with the budget constraint gives

$$x_1^* = \frac{a}{a+b} \frac{m}{p_1}, \quad x_2^* = \frac{b}{a+b} \frac{m}{p_2}.$$

With Cobb–Douglas preferences, the expenditure share of each good is constant and independent of prices and income:

$$\frac{p_1 x_1^*}{m} = \frac{a}{a+b}, \quad \frac{p_2 x_2^*}{m} = \frac{b}{a+b}.$$

Implications of the MRS condition

Common prices and the MRS

If consumers face the same prices in a market, have smooth preferences and both choose an interior optimum, then their marginal rate of substitution must be the same:

$$MRS_A = -\frac{p_1}{p_2} = MRS_B.$$

Even though they may consume very different quantities of goods 1 and 2, they have the same marginal willingness to trade one good for the other at their chosen bundles.