LECTURE 10

Cost Curves

5SSPP217 – Intermediate Microeconomics

Recap: the cost of a specified output

The **cost function** gives the lowest possible cost of producing a specific level of output:

$$c(w_1, w_2, y) = w_1 x_1^c(w_1, w_2, y) + w_2 x_2^c(w_1, w_2, y)$$

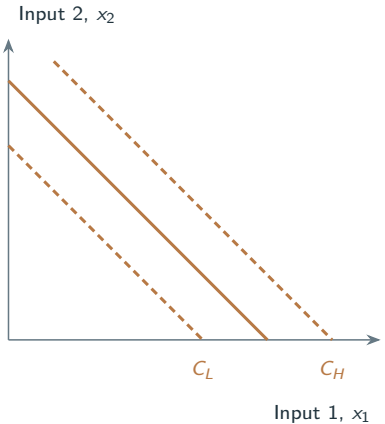
Assuming that input prices are fixed, we can write the cost function simply as

$$c(y)$$

Recap: isocost lines and tangency

An **isocost line** shows input bundles with the same cost C:

$$w_1 x_1 + w_2 x_2 = C.$$



Recap: isocost lines and tangency

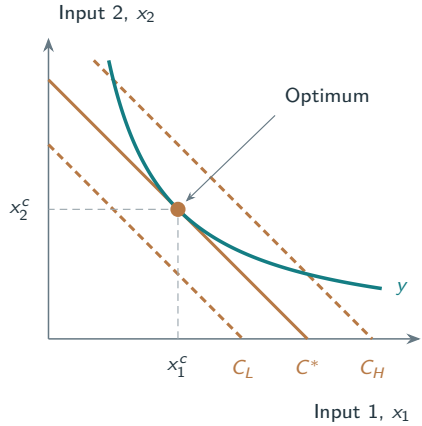
An **isocost line** shows input bundles with the same cost C:

$$w_1 x_1 + w_2 x_2 = C.$$

For a given level of output, a firm will choose the lowest isocost line. If the interior optimum is smooth:

$$-\frac{MP_1}{MP_2} = -\frac{w_1}{w_2} \text{or } \frac{MP_1}{w_1} = \frac{MP_2}{w_2}.$$

The last pound spent on either input produces the same extra output.



Recap: average cost and returns to scale

The **average cost function** measures cost per unit of output:

$$AC(y) = \frac{c(y)}{y}, \qquad y > 0.$$

Suppose all inputs are multiplied by $t > 1$, the effect on output depends on the returns to scale

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Suppose all inputs are multiplied by $t > 1$, the effect on output depends on the returns to scale

Returns to scale	**Total cost**	**Average cost**
Constant	$c(ty) = t\,c(y)$	$AC(ty) = AC(y)$
Increasing	$c(ty) < t\,c(y)$	$AC(ty) < AC(y)$
Decreasing	$c(ty) > t\,c(y)$	$AC(ty) > AC(y)$

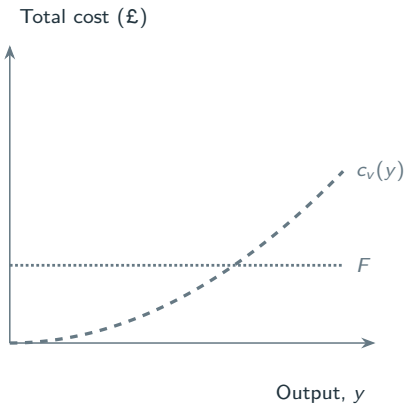
Total, average and marginal costs

Total, fixed and variable costs

By definition, some inputs (and their costs) are fixed in the **short run**.

Total cost can be written as the sum of variable costs $c_v(y)$ and fixed costs F:

$$c(y) = c_v(y) + F.$$



Total, fixed and variable costs

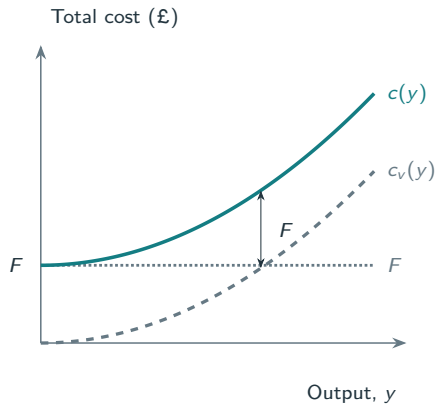
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$$c(y) = c_v(y) + F.$$

At each level of output, total cost lies exactly F above variable cost.

When $y = 0$, $c_v(0) = 0$ and total cost is equal to F.



Average costs

Let's define average variable cost $AVC(y)$ and average fixed cost $AFC(y)$ as

$$AC(y) = \frac{c(y)}{y}, \qquad AVC(y) = \frac{c_v(y)}{y}, \qquad AFC(y) = \frac{F}{y}.$$

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Then if we split total cost into its variable and fixed components we can express average cost as the sum of average variable and average fixed cost:

$$AC(y) = \frac{c_v(y) + F}{y} = \frac{c_v(y)}{y} + \frac{F}{y} = AVC(y) + AFC(y).$$

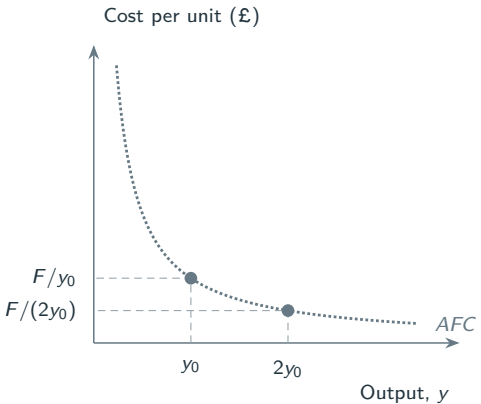
Average fixed costs

Since fixed costs F do not depend on output, increasing output spreads the same fixed payment over a larger output:

$$AFC(y) = \frac{F}{y},$$

$$\lim_{y \to \infty} AFC(y) = 0.$$

If we reduce output towards zero, average fixed costs go to infinity



Average fixed costs

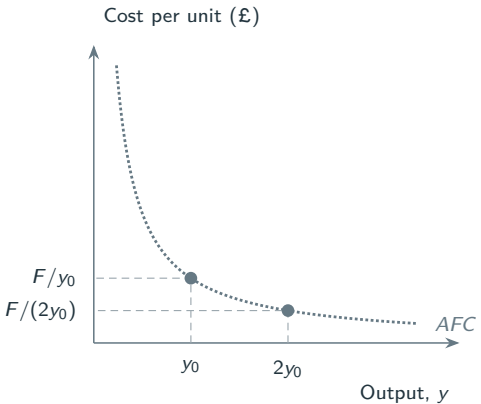
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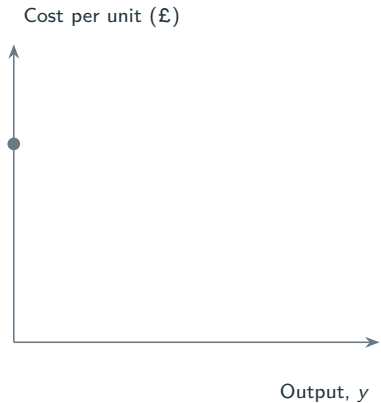
Average variable costs

If the firm produces one unit of output, its average variable cost is $AVC(1) = c_v(1)$.

Suppose output increases, what happens to $AVC(y)$?

It can either rise, if fixed factors have to be stretched across output, or fall, if production can be organised more efficiently.

Eventually, the former effect often dominates so if output is sufficiently high, average variable costs go up.



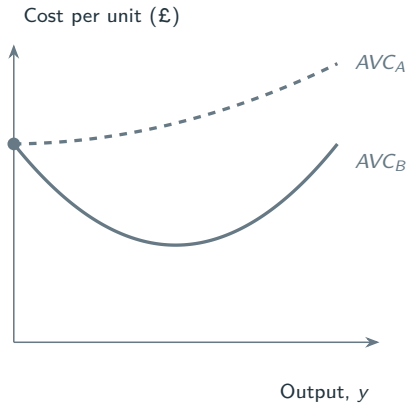
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Average cost: adding AFC and AVC

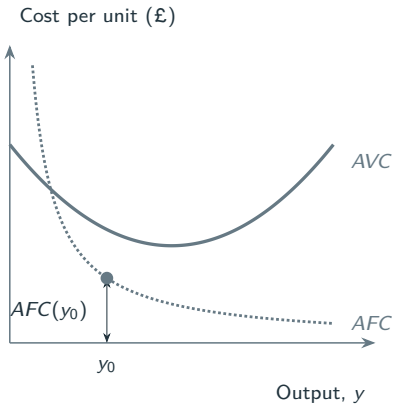
We obtain the average cost curve by summing the AFC and the AVC curves.

$$AC(y) = AVC(y) + AFC(y).$$

If average variable costs fall, or rise by less than the fall in average fixed costs, the AC curve initially falls.

The eventual increase in AVC ultimately dominates, so that the AC curve goes up for high levels of output.

The vertical gap between AC and AVC equals AFC. This gap shrinks as output increases.



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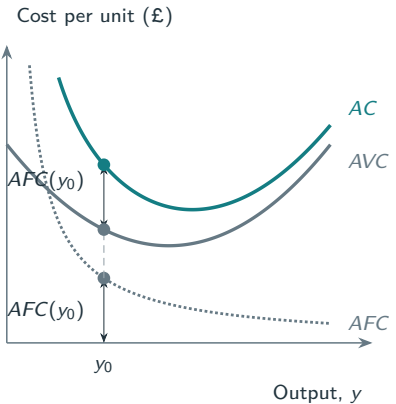
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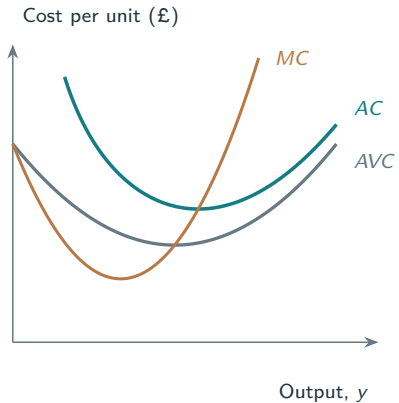
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Marginal cost

For a small output increase, the cost change per unit is approximately $MC(y)$:

$$\frac{c(y + \Delta y) - c(y)}{\Delta y}$$



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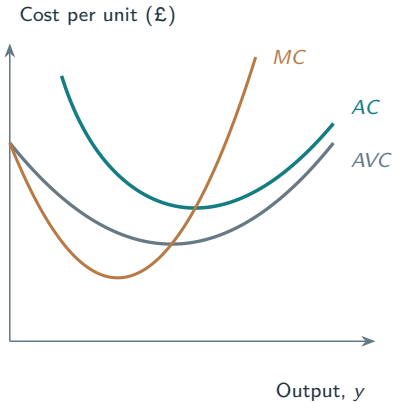
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Fixed cost is $c_f(y) = F$, so

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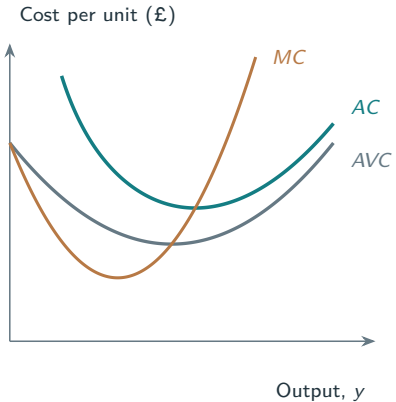
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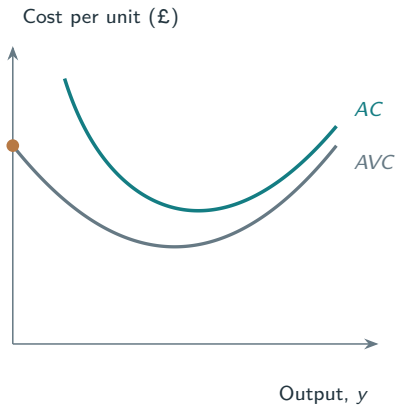
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Drawing the marginal cost curve

The first unit costs

$$MC(1) = c_v(1) - c_v(0) = AVC(1).$$

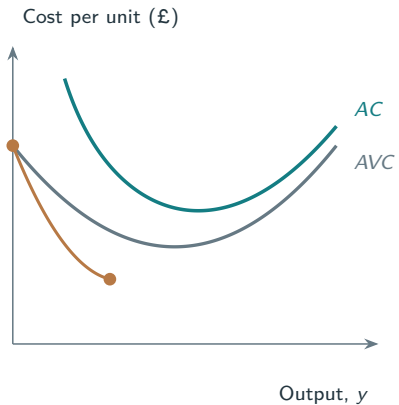


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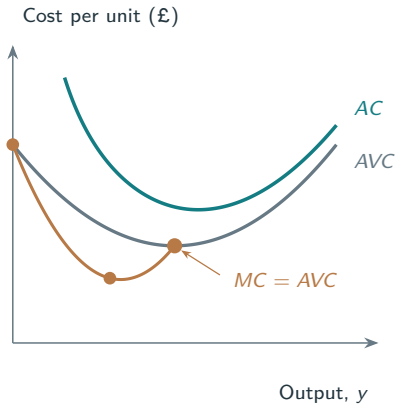
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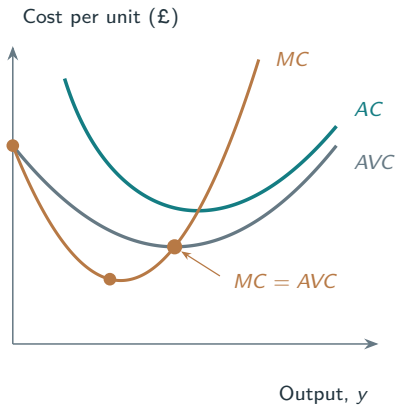
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As AVC rises, $MC > AVC$.

Marginal cost also passes through the minimum of AC.



How marginal cost changes an average

An additional unit that costs **less** than the current average pulls the average down. An additional unit that costs **more** pushes it up.

Suppose 10 units cost £50, so average cost is £5.

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If the next unit costs £4, the new average is

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If instead it costs £6, the new average is

$$\frac{50+6}{10+1} = \frac{56}{11} > 5.$$

Long-run cost

Long-run cost: choosing plant size

In the short-run, at least one factor is fixed. Suppose that factor is plant size k, then the short-run cost function is

$$c_s(y; k)$$

Suppose we could choose k and let $k(y)$ denote the cost-minimising plant size for a given level of output y. In the long-run, we can choose this cost-minimising level of k, so the cost function in the long run is

$$c(y) = c_s(y; k(y)).$$

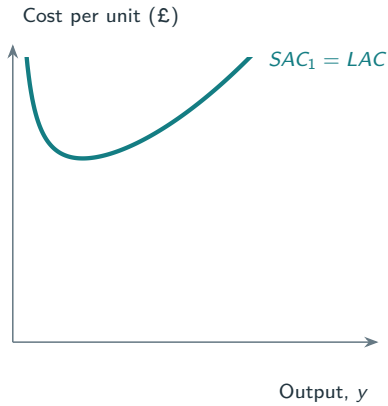
Let k^* be the optimal level of k for some level of output y^*, then the long-run cost is always equal to or less than the short-run cost

$$c(y) \leq c_s(y; k^*).$$

Discrete plant sizes

Suppose that only plant size k_1 is available, even in the long run. Then,

$$LAC(y) = SAC(y; k_1).$$

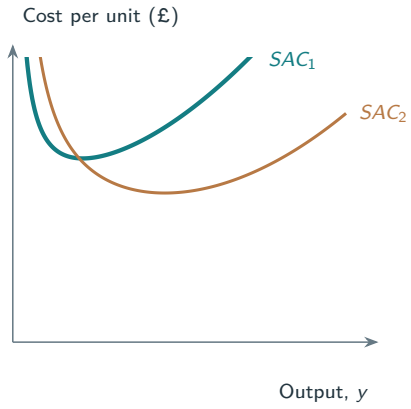


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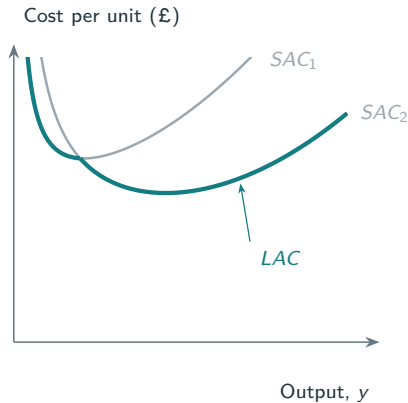
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The long-run average cost function chooses k such that costs are lowest:

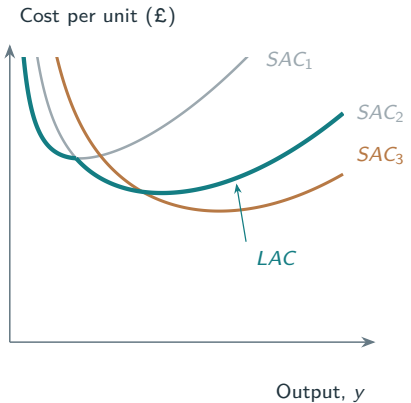
$$LAC(y) = \min\{SAC(y; k_1), SAC(y; k_2)\}.$$

The long-run curve follows the **lower parts of the two curves**



Discrete plant sizes

Now suppose a third plant size, k_3, is available. This gives the firm another option at each output.



Discrete plant sizes

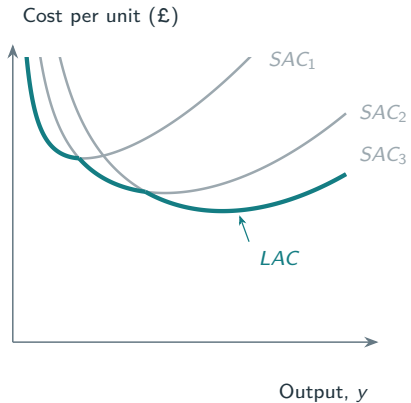
Now suppose a third plant size, k_3, is available. This gives the firm another option at each output.

The firm considers all plants and chooses the cheapest:

$$LAC(y) = \min_{i=1,2,3} SAC(y; k_i).$$

The long-run curve now follows the **lower parts of the three curves**.

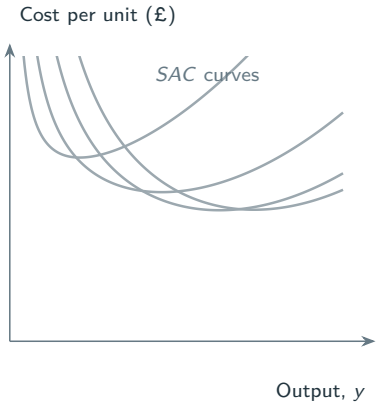
The additional choice can lower cost, but cannot raise it.



Long-run average cost: the lower envelope

Suppose now that there is a continuum of plant sizes available to the firm.

In the long run, the firm can always choose the plant size with the lowest average costs for a given level of output.



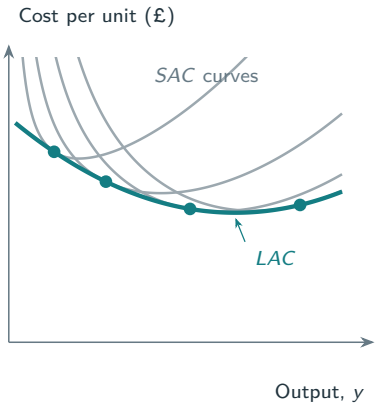
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The long-run average cost function becomes:

$$LAC(y) = \min_k SAC(y; k).$$



Short-run and long-run average cost

Plant size k^* is defined as the cheapest plant for output level y^*. Then

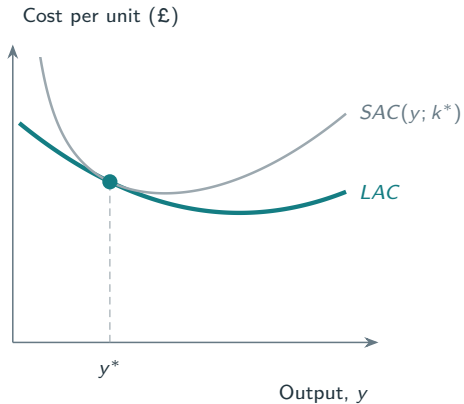
$$SAC(y^*, k^*) = LAC(y^*).$$

There may be a different output level for which $SAC(y, k^*)$ are even lower:

$$SAC(y_{\min}, k^*) < SAC(y^*, k^*).$$

Importantly, this point does not need to be part of the LAC. It may be possible to lower costs at $y_{\min}$ if we change k away from k^*

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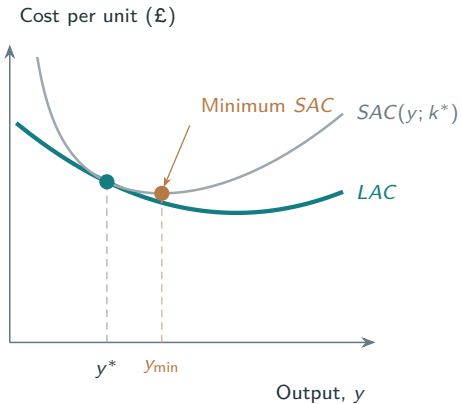
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A kitchen example

A restaurant has to choose whether to use a small kitchen or a large kitchen.

For a modest order (y_{small}), the small kitchen may be cheaper $k^*_{\text{small}} = k(y_{\text{small}})$: its lower fixed costs outweigh the labour savings offered by the larger kitchen.

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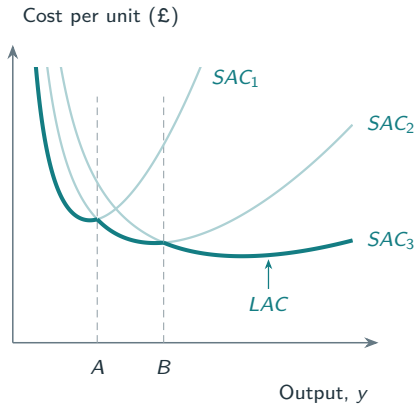
But for that **larger order**, the large kitchen might be cheaper still: more space and equipment could save enough labour to outweigh its higher fixed costs ($LAC(y_{\text{large}}) = SAC(y_{\text{large}}, k^*_{\text{large}}) < SAC(y_{\text{large}}, k^*_{\text{small}})$).

Long-run cost: choosing the plant

With discrete plant sizes, which plant should the firm use at each output?

For a given level of output y, the firm will choose the plant size that minimises (total and) average cost:

Output	**Chosen plant**
$y < A$	k_1
$A < y < B$	k_2
$y > B$	k_3



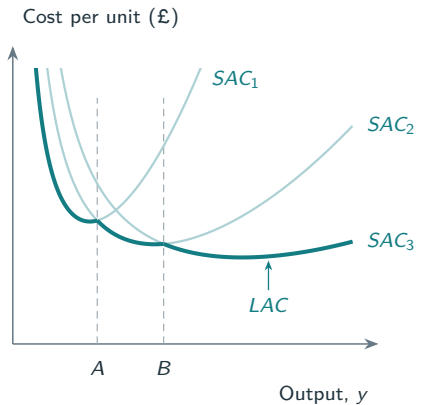
Long-run marginal cost

As long as y remains below A, the chosen plant remains k_1. The marginal cost MC_1 is therefore the long-run marginal cost for $y < A$.

Similarly for $A < y < B$ and $y > B$:

Output	**LMC**
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$A < y < B$	
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LMC is not generally the lower envelope of the *MC* curves.



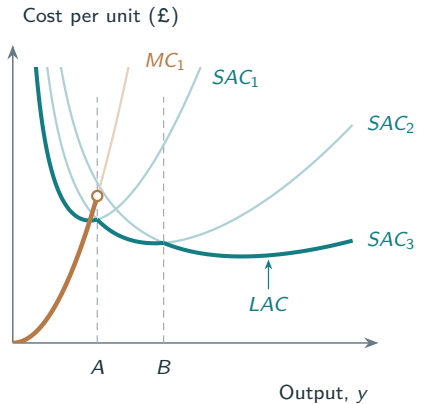
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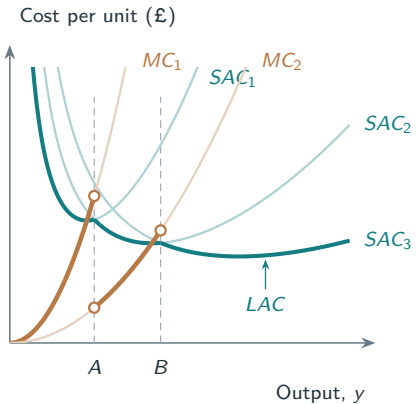
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As long as y remains below A, the chosen plant remains k_1. The marginal cost MC_1 is therefore the long-run marginal cost for $y < A$.

Similarly for $A < y < B$ and $y > B$:

Output	**LMC**
$y < A$	$MC_1(y)$
$A < y < B$	$MC_2(y)$
$y > B$	

LMC is not generally the lower envelope of the *MC* curves.



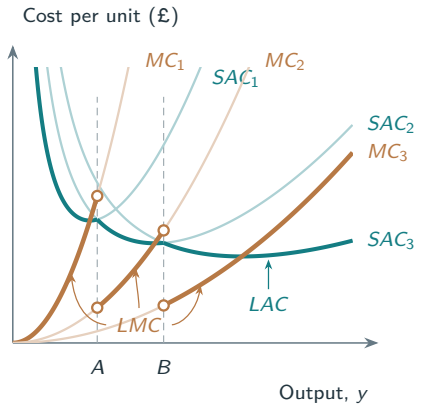
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Output	**LMC**
$y < A$	$MC_1(y)$
$A < y < B$	$MC_2(y)$
$y > B$	$MC_3(y)$

LMC is not generally the lower envelope of the *MC* curves.



The upcoming problem set

The take-home problem set covers Lectures 1–12 and counts for 30% of the module mark.

Release date: 13 November 2026
Submission deadline: 20 November 2026, 15:00

To prepare:

- Read the textbook!
- Revisit the seminar exercises and work through the solutions yourself.
- Try additional exercises by Theodore Bergstrom.