

LECTURE 1

Constraints and preferences

Intermediate Microeconomics

PART 1

Budget constraint

Consumption bundles and prices

A consumer's **consumption bundle** is denoted by:

$$X = (x_1, x_2, \dots, x_n),$$

where x_i is the quantity of good i consumed.

The corresponding prices are denoted by:

$$P = (p_1, p_2, \dots, p_n),$$

where p_i is the price of good i .

The budget constraint

Given prices $P = (p_1, p_2, \dots, p_n)$ and income m , the **budget constraint** can be written as:

$$p_1x_1 + p_2x_2 + \dots + p_nx_n \leq m$$

The consumer's *affordable* consumption bundles, also known as the **budget set**, are those that do not cost more than m .

From n to 2

We can simplify our analysis by interpreting x_2 as a **composite good** that stands for the amount of money that the consumer can spend on all goods other than good 1.

Since the price of a unit of money is 1, the budget constraint becomes

$$p_1 x_1 + x_2 \leq m$$

In general, when we normalise the price of one good to 1, we call that good the **numeraire** good.

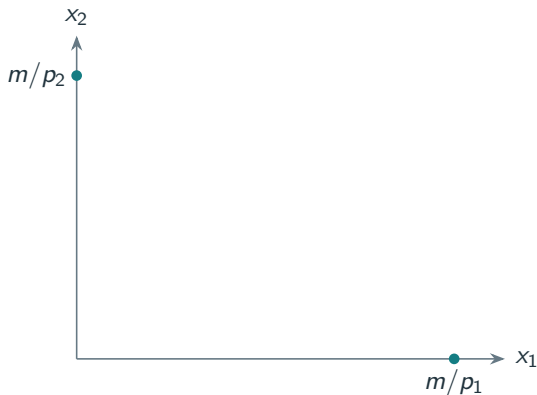
The budget line and budget set

The **budget line** is the set of bundles that cost exactly m :

$$p_1x_1 + p_2x_2 = m.$$

If a consumer consumes zero of good 1 and spends all of their income on good 2, they consume m/p_2 of it.

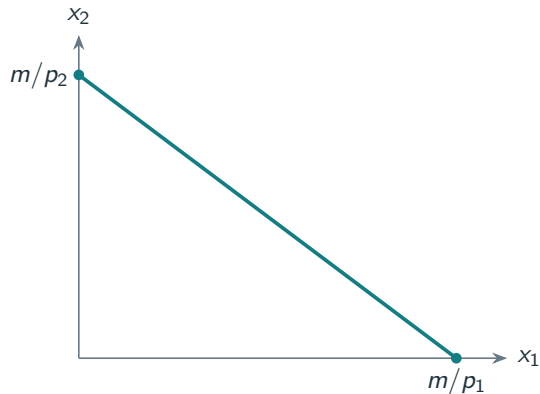
If a consumer consumes zero of good 2 and spends all of their income on good 1, they consume m/p_1 of it.



The budget line

The line joins the two intercepts:

$$\frac{m}{p_1} \quad \text{and} \quad \frac{m}{p_2}.$$

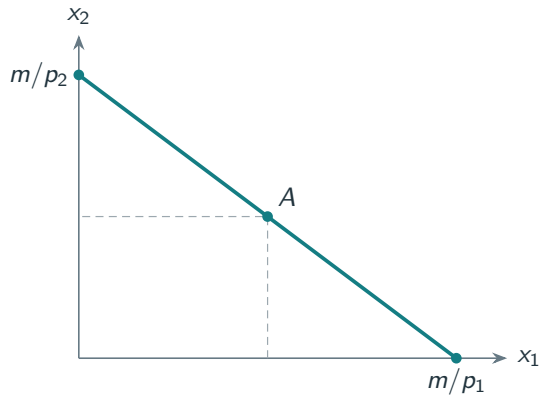


The budget line

Bundle A lies on the budget line.

$$p_1x_1 + p_2x_2 = m$$

Just affordable

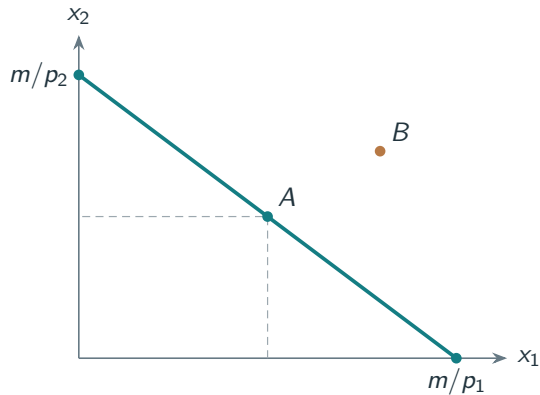


The budget line

Bundle B lies outside the budget line.

$$p_1x_1 + p_2x_2 > m$$

Unaffordable

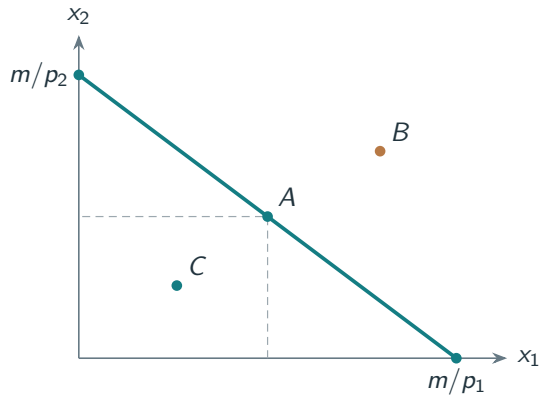


The budget line

Bundle C lies inside the budget line.

$$p_1x_1 + p_2x_2 < m$$

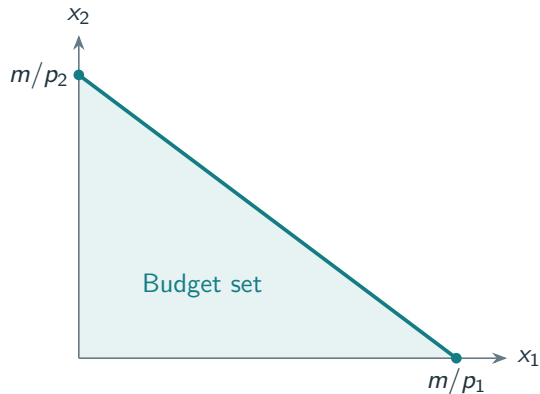
Affordable



The budget line

Every bundle on or below the budget line belongs to the **budget set**.

$$p_1x_1 + p_2x_2 \leq m$$



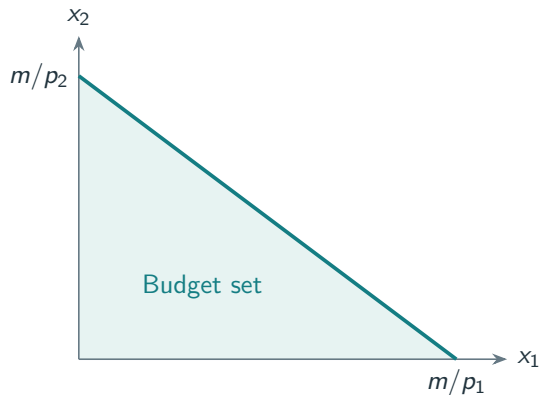
The slope of the budget line

Rearranging the budget constraint equation gives

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1.$$

The slope of the budget line is thus

$$-p_1/p_2$$

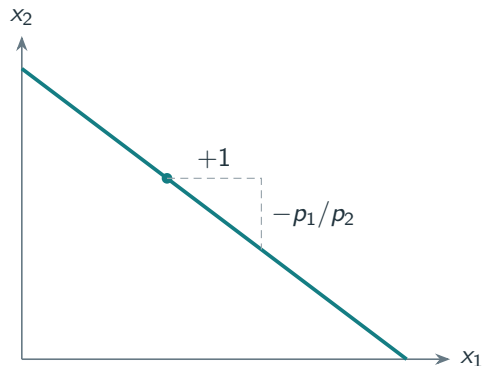


The opportunity cost of good 1

The slope of the budget line measures the trade-off between consumption of good 1 and good 2: one extra unit of good 1 costs

$$\frac{p_1}{p_2}$$

units of good 2.

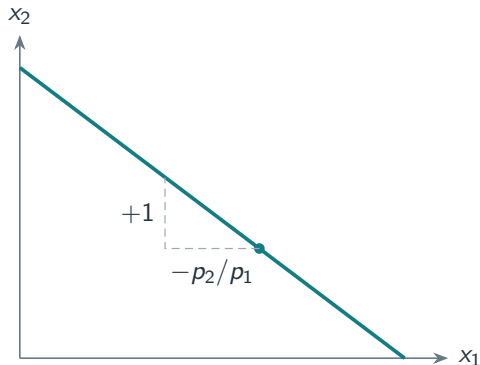


The opportunity cost of good 2

One extra unit of good 2 costs

$$\frac{p_2}{p_1}$$

units of good 1.



Income and price changes

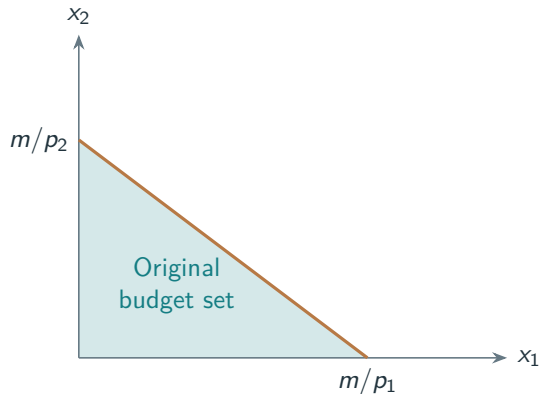
The budget set depends on both **prices** and **income**.

- What happens when income changes?
- What happens when the price of one good changes?

An increase in income

Hold prices constant.

How does the budget set change when income rises from m to $m' > m$?

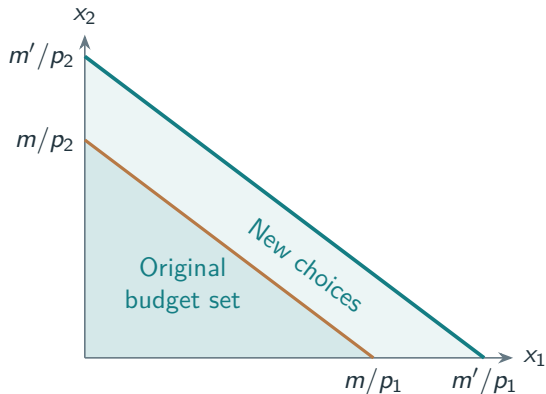


An increase in income

Both intercepts increase.

The new budget line is parallel to the original line.

$$\text{slope} = -\frac{p_1}{p_2}$$



Income changes

When keeping prices fixed, an increase in income shifts the budget line in parallel.

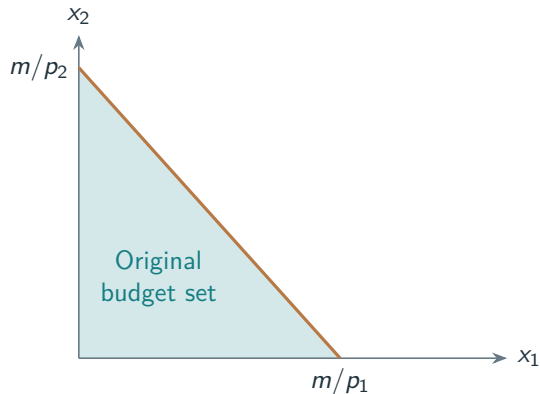
- Higher income: an outward shift and a larger budget set.
- Lower income: an inward shift and a smaller budget set.

A fall in the price of good 1

Suppose income is held constant and the price of good 1 falls:

$$p'_1 < p_1.$$

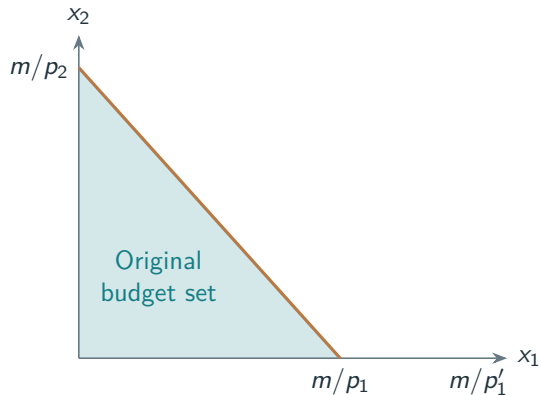
What happens to the intercepts and slope?



A fall in the price of good 1

The good 2 intercept remains m/p_2 .

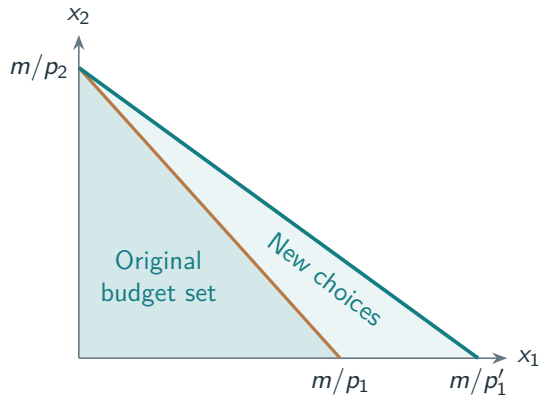
The good 1 intercept moves from m/p_1 to m/p'_1 .



A fall in the price of good 1

The line pivots outward and becomes flatter.

$$\text{slope} = -\frac{p_1}{p_2} \longrightarrow -\frac{p'_1}{p_2}$$

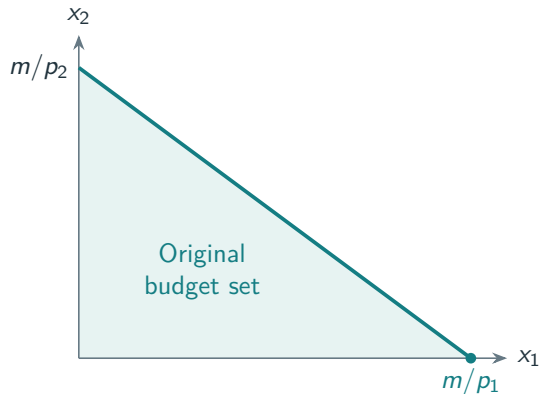


An increase in the price of good 2

Suppose income is held constant and the price of good 2 rises:

$$p'_2 > p_2.$$

What happens to the budget line?



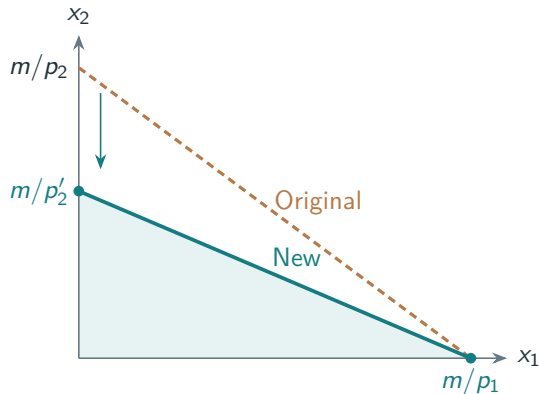
An increase in the price of good 2

The good 2 intercept falls from m/p_2 to m/p'_2 .

The good 1 intercept stays at m/p_1 .

The line pivots inward and becomes flatter:

$$\text{slope} = -\frac{p_1}{p'_2} > -\frac{p_1}{p_2}.$$



Is the budget line straight?

So far, we have assumed that p_1 and p_2 are constants and do not depend on x_1 and x_2 .

As a result, the budget line has a constant slope: $-\frac{p_1}{p_2}$

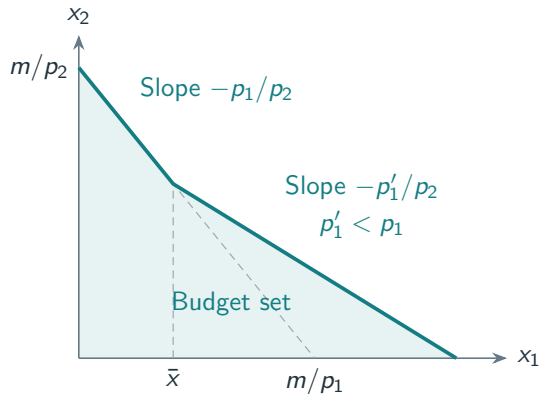
In reality, unit prices may depend on quantities consumed (bulk-buying discounts; quantity penalties).

As a result, the budget line may be curved or kinked.

The budget set with a quantity discount for good 1

If there is a quantity discount for good 1 such that p_1 is lower for $x > \bar{x}$, the slope is flatter to the right of $\bar{x}$:

$$-\frac{p_1}{p_2} < -\frac{p'_1}{p_2}$$



PART 2

Preferences

Consumer preferences

The consumer can rank any two consumption bundles (x_1, x_2) and (y_1, y_2) .

The symbol $\succ$ means that one bundle is **strictly preferred** to another:

$$(x_1, x_2) \succ (y_1, y_2)$$

.

The symbol $\sim$ means that the consumer is **indifferent** between two bundles of goods:

$$(x_1, x_2) \sim (y_1, y_2)$$

.

The symbol $\succeq$ means that the consumer prefers or is indifferent between the two bundles, or in other words **weakly prefers** (x_1, x_2) to (y_1, y_2) :

$$(x_1, x_2) \succeq (y_1, y_2)$$

.

The logic of consumer preferences

If $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (x_1, x_2)$, then

$$(x_1, x_2) \sim (y_1, y_2)$$

.

If $(x_1, x_2) \succeq (y_1, y_2)$ and it is not the case that $(x_1, x_2) \sim (y_1, y_2)$, then

$$(x_1, x_2) \succ (y_1, y_2)$$

.

Axioms of consumer theory

Economists make three fundamental assumptions about preferences:

Completeness. Any two bundles can be compared. Given any x-bundle and y-bundle, it must be that $(x_1, x_2) \succeq (y_1, y_2)$, $(y_1, y_2) \succeq (x_1, x_2)$ or both.

Reflexivity. Any bundle is at least as good as itself: $(x_1, x_2) \succeq (x_1, x_2)$.

Transitive. If $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (z_1, z_2)$, then it must be that $(x_1, x_2) \succeq (z_1, z_2)$.

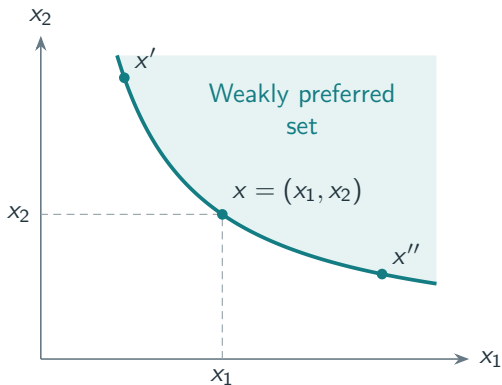
An indifference curve

Given a certain consumption bundle (x_1, x_2) , the set of consumption bundles that is weakly preferred to (x_1, x_2) is called the **weakly preferred set**.

The bundles on the boundary of this set is the **indifference curve**.

Every bundle on the same curve gives the same preference ranking.

$$x' \sim x \sim x''$$

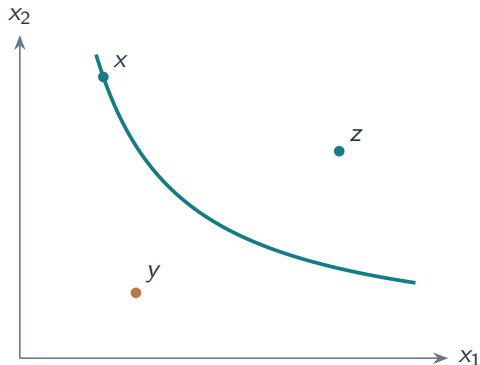


Ranking bundles

For the preferences illustrated:

$$z \succ x \succ y$$

Bundle z lies on a higher indifference curve than x . Bundle y lies on a lower one.



Indifference curves cannot intersect

Suppose I_1 and I_2 intersect at x , then

$$x \sim y \quad \text{and} \quad x \sim z$$

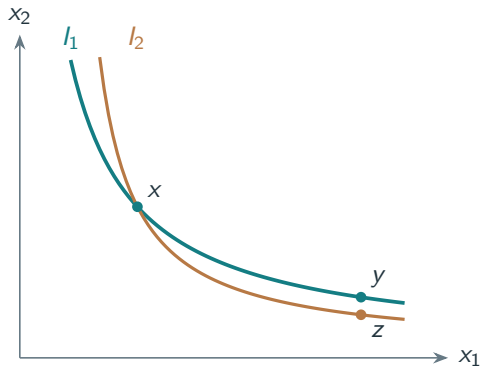
Transitivity then implies

$$y \sim z.$$

but we know that

$$y \succ z,$$

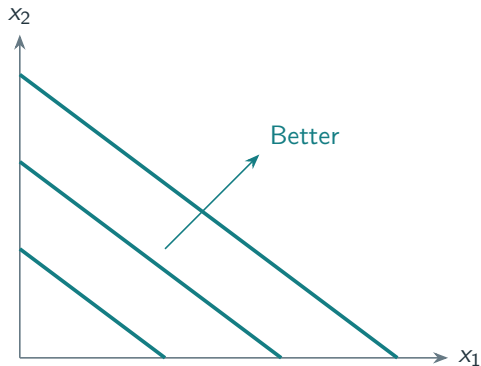
so this situation cannot occur.



Perfect substitutes

Two goods are **perfect substitutes** if the consumer is willing to exchange one good for the other at a constant rate.

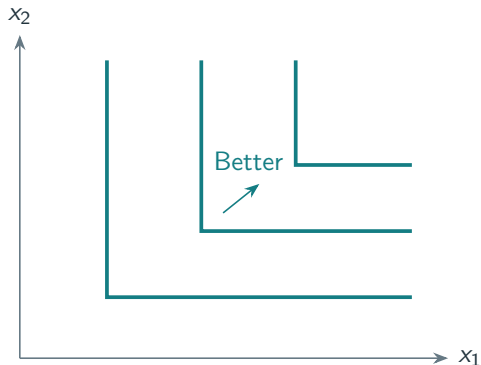
Indifference curves are parallel straight lines.



Perfect complements

Goods that are always consumed together in fixed proportions are **perfect complements**.

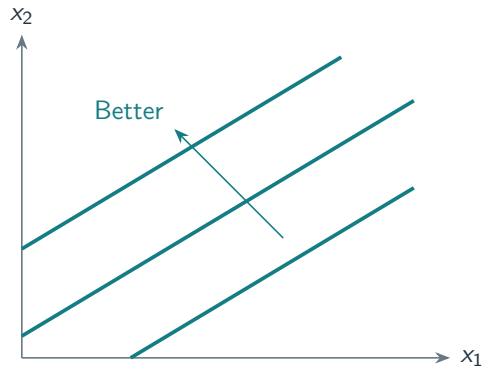
Consuming more of one good without increasing consumption of the other good leaves the consumer equally well off.



Bads

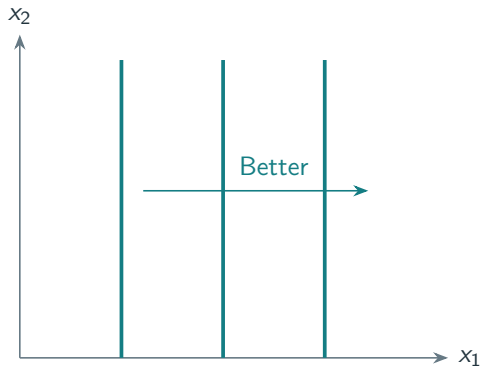
A **bad** is a commodity that the consumer doesn't like, x_1 here.

If a consumer consumes more of the bad, they have to be compensated by higher consumption of the other good to remain indifferent.



Neutrals

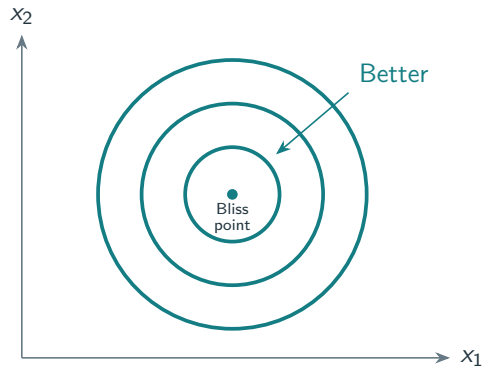
A **neutral good**, x_2 here, is a good where the consumer is indifferent between consuming it or not.



Satiation

The consumer has a most-preferred bundle: the **bliss point**.

Bundles closer to the bliss point are preferred over those further away.



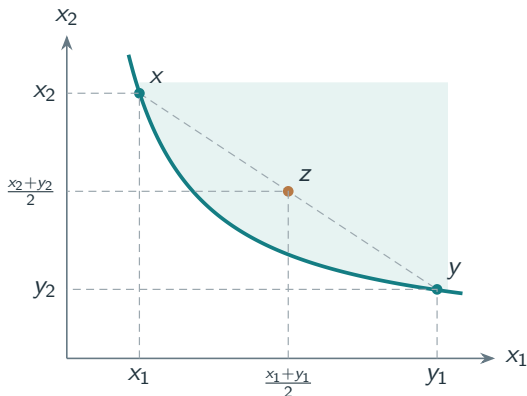
Well-behaved preferences

Preferences are **well-behaved** if they satisfy:

Monotonicity. More of either good is desirable. To remain indifferent, increasing the consumption of one good must come at the expense of the other.

Convexity. Consumers prefer averages over extremes:

$$z \succeq x$$

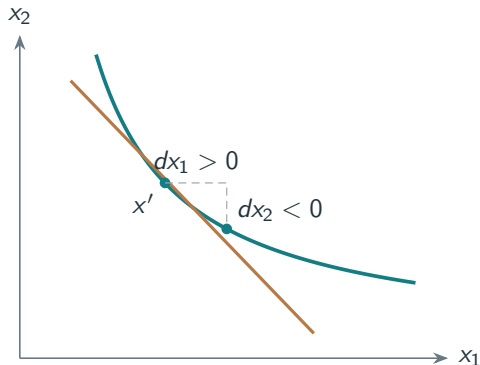


Marginal rate of substitution

The slope of the indifference curve is known as the **marginal rate of substitution**.

It measures how much of one good must be given up in order to increase consumption of the other good by one unit and remain equally well-off.

$$MRS = \frac{dx_2}{dx_1}$$



Representing preferences

The budget constraint and preferences are the foundations of consumer choice.

Assuming monotonicity and convexity makes them easier to analyse.

Next lecture, we represent preference rankings with **utility functions**.