

5SSPP217 Microeconomics
Week 6 exercises
Cost minimisation and cost curves

1. Firm A has production function

$$f(x_1, x_2) = \sqrt{x_1 x_2}, \quad x_1, x_2 \geq 0.$$

Input prices are $w_1 = 8$ and $w_2 = 2$ monetary units per unit of input. The firm must produce a target output $Y > 0$.

- (a) Draw the isoquant for Y and several isocost lines. Identify the cost-minimising input bundle.
- (b) Find the cost-minimising input bundle analytically. Explain the intuition behind the optimality condition and derive the cost function $c(Y)$.

2. A firm has production function

$$f(x_1, x_2) = \min\{4x_1, x_2\}, \quad x_1, x_2 \geq 0,$$

and input prices $w_1, w_2 > 0$. It must produce a target output $Y > 0$.

- (a) Draw the isoquant for Y and several isocost lines. Identify the cost-minimising input bundle.
- (b) Find the cost-minimising input bundle analytically and derive the cost function $c(w_1, w_2, Y)$.

3. A firm has production function

$$f(L, K) = L + 2K, \quad L, K \geq 0,$$

where L is labour and K is capital. Input prices are $w_L, w_K > 0$, and the target output is $Y > 0$.

- (a) Draw the isoquant for Y and isocost lines for different relative input prices. Identify the cost-minimising input bundles in each case.
- (b) Find all cost-minimising input bundles and the cost function $c(w_L, w_K, Y)$ for every value of w_L/w_K , including corner solutions and any case with multiple optima.

4. A firm's short-run total cost function is

$$c(y) = 4000 + 5y + 10y^2, \quad y \geq 0.$$

Output is continuously divisible, and costs are measured in monetary units. For average costs, restrict attention to $y > 0$.

- (a) Write an expression for each of the following and draw the corresponding curves. Distinguish total costs from costs per unit of output when labelling the graphs.
- Total fixed cost.
 - Average fixed cost.
 - Total variable cost.
 - Average variable cost.
 - Average total cost.
 - Marginal cost.
- (b) Find the output that minimises average total cost. Show how marginal cost compares with average total cost below, at and above this output.

5. Domination Pizzas has short-run total cost

$$C(y, K) = \frac{17y^2}{32K} + 0.25K,$$

where $y \geq 0$ is pizza output and $K > 0$ is oven capacity, measured in oven equivalents. Treat output and capacity as continuously divisible. Costs, including capacity lease payments, are measured in monetary units for the same production period. Capacity is currently fixed at $K = 4$.

- (a) Calculate the average total cost of producing 10 pizzas.
- (b) The manager is considering leasing 10 additional oven equivalents. Calculate the average total cost of producing 10 pizzas with this capacity. Is it higher or lower than with $K = 4$? Explain.
- (c) Find the long-run cost function $c(y)$ for $y > 0$, allowing capacity to vary. Assuming the underlying production function is homogeneous, determine whether the technology exhibits increasing, constant or decreasing returns to scale.