

5SSPP217 Microeconomics
Week 5 exercises
Technology and profit maximisation

1. Choose how many hours to study for each of three exams. Study time is chosen separately for each subject, in whole hours from 0 to 10. The table gives the exam marks obtained; zero hours of study yield zero marks in each subject.

Hours	Subject A	Subject B	Subject C
1	40	60	29
2	65	90	46
3	80	100	60
4	90	100	72
5	95	90	82
6	99	75	90
7	100	55	96
8	100	33	100
9	99	8	100
10	95	0	100

- (a) Compute the marginal product of each hour of study for each subject.
 - (b) Assess whether the marginal product of study time diminishes in each subject. Identify any exceptions and suggest an explanation for the pattern.
 - (c) Each hour of study has an opportunity cost equivalent to 12 exam marks. Find all study times that maximise exam marks net of this cost for each subject, and the resulting exam marks.
2. Characterise the returns to scale of the following production functions for capital $K > 0$ and labour $L > 0$. For parts (b), (c) and (d), also determine whether the marginal product of each input diminishes as that input increases, holding the other input fixed.
 - (a) $y = a_1L + a_2K$, where $a_1, a_2 > 0$.
 - (b) $y = 3K + L^2$.
 - (c) $y = K^{2/3}L^{2/3}$.
 - (d) $y = K^{1/4}L^{1/2}$.
 - (e) $y = \min\{a_1K, a_2L\}$, where $a_1, a_2 > 0$.
 - (f) $y = (\min\{K, L\})^{1/2}$.

3. Sunrise Ltd. uses labour $L \geq 0$, measured in hours, to produce widgets:

$$y = AL, \quad A > 0.$$

The firm takes the wage $w > 0$ per labour hour and the output price $p > 0$ per widget as given. It can choose any non-negative amount of labour and has no other costs. Characterise its profit-maximising labour demand and output supply for all price configurations.

4. Acme Container Co. produces egg cartons using

$$y = 25L^{0.6}K^{0.3}, \quad K, L \geq 0,$$

where y is measured in lots of 1,000 cartons, L in person-hours and K in machine-hours. The wage is £10 per person-hour and the capital rental price is £25 per machine-hour. Acme takes the output price $p > 0$, measured in pounds per lot of 1,000 cartons, as given. Both inputs are continuously divisible and variable, and there are no other costs.

- (a) Determine the capital–labour ratio that maximises Acme’s profits.
- (b) Write Acme’s profit function and derive its profit-maximising input choices K^* and L^* as functions of p .

5. A price-taking firm has production function

$$y = \min\{x_1, \sqrt{x_2}\}, \quad x_1, x_2 \geq 0.$$

Both inputs are continuously divisible and variable. Their unit prices are $w_1, w_2 > 0$, the output price satisfies $p > w_1$, and there are no other costs.

- (a) Characterise returns to scale by comparing $f(tx_1, tx_2)$ with $tf(x_1, x_2)$ for $t > 1$ and positive inputs. Distinguish strict inequalities from equality where appropriate.
- (b) Find the profit-maximising inputs x_1^*, x_2^* and output y^* as functions of p, w_1, w_2 .

6. A price-taking firm has production function

$$y = 3x_1 + x_2, \quad x_1, x_2 \geq 0.$$

Inputs are continuously divisible, their unit prices are $w_1, w_2 > 0$, the output price is $p > 0$, and there are no other costs. Characterise all optimal choices.

- (a) In the short run, x_2 is fixed at 2 and its cost $2w_2$ must be paid. Write the short-run profit function and characterise optimal x_1 as a function of w_1/p .
- (b) In the long run, both inputs are variable and may take any non-negative values. Characterise the profit-maximising inputs and output for all configurations of p, w_1, w_2 .