

5SSPP217 Microeconomics
Week 4 exercises
Intertemporal choice and uncertainty

1. A consumer lives for two periods and regards current and future consumption, $c_1, c_2 \geq 0$, as normal goods. The consumer has fixed endowments $m_1, m_2 > 0$, measured in units of consumption, and can borrow or save at the same real interest rate $r > -1$.
 - (a) Suppose the interest rate rises to $r' > r$. Draw the change in the budget constraint, with c_1 on the horizontal axis. Explain how the substitution effect changes current and future consumption.
 - (b) If the consumer was saving before the change, how does the income effect change current and future consumption?
 - (c) Repeat part (b) for a consumer who was borrowing before the change.
2. Frank the farmer earns £100 in each of two periods. Savings earn 20% interest, whereas borrowing costs 10%. Measure consumption $c_1, c_2 \geq 0$ in pounds, with no change in consumption prices between periods.
 - (a) Frank may either save or borrow, but cannot do both simultaneously. Draw his budget constraint, identifying both intercepts, the slopes and the endowment point.
 - (b) Frank can now borrow and lend simultaneously as an informal banker, lending his own funds and borrowed funds to other farmers at 20% interest. All these loans are repaid with certainty in period 2. The repayment of Frank's borrowing (including interest) cannot exceed his period-2 farming income. Draw his new budget constraint, identifying its intercepts and slope.
 - (c) Frank's preferences are represented by

$$u(c_1, c_2) = c_1 c_2.$$

Find his optimal consumption bundle in each of parts (a) and (b).

3. In an isolated mountain village, corn is the only crop. Each villager harvests 100 bushels in period 1 and nothing in period 2, with no initial stocks. Villagers can only consume or store their own corn. Corn is continuously divisible, and rats eat 25% of the corn stored between periods. Each villager has utility

$$u(c_1, c_2) = c_1 c_2,$$

where $c_1, c_2 \geq 0$ are bushels consumed in each period.

- (a) Draw the budget constraint and mark the endowment point. Use the Lagrange method to find the optimal intertemporal consumption bundle.
 - (b) Introducing cats at no cost reduces the proportion of stored corn eaten by rats to 10%. Draw the new budget constraint and find the optimal bundle analytically. Explain whether villagers are better or worse off, using the change in consumption possibilities rather than calculating utility levels.
4. Sam owns a farm in Dorset and maximises expected utility. His Bernoulli utility over wealth is

$$u(w) = 15\sqrt{w}, \quad w \geq 0,$$

where w is measured in pounds.

- (a) Under certainty, any strictly increasing transformation of a utility function represents the same preferences. Does transforming Sam's Bernoulli utility $u(w)$ before taking expectations necessarily preserve his preferences over lotteries?
 - (b) Without insurance, Sam's wealth is £360,000 if there is no flood and £160,000 if there is a flood. The flood probability is $1/15$. Find his certainty equivalent, his risk premium and the maximum total premium he would pay for full flood insurance.
5. Ahmed has £1,000 and may invest any amount $x \in [0, 1000]$ in a start-up. Success yields a net return of 20% on the amount invested; failure loses 10% of that amount. The probability of success is $p \in [0, 1]$. Uninvested funds earn zero interest. Ahmed has no other wealth or income and cannot borrow. He spends all his final wealth on consumption at £1 per unit and maximises expected utility with Bernoulli utility

$$u(c) = c^2, \quad c \geq 0.$$

- (a) Find the state-contingent consumption bundles when Ahmed invests nothing and when he invests all £1,000. Derive and draw the feasible segment of his state-contingent budget line, with consumption in the failure state on the horizontal axis. Discuss the interpretation of its slope.
- (b) Find all optimal investment amounts as a function of p , including boundary choices and indifference. Illustrate the choices using the budget line and indifference curves.