

5SSPP217 Microeconomics
Week 3 solutions
Consumer demand

1. (a) For perfect substitutes, the consumer buys the cheaper good. The optimal bundles are

$$(x_1^*, x_2^*) = \begin{cases} (m/p_1, 0), & p_1 < p_2, \\ (0, m/p_2), & p_1 > p_2. \end{cases}$$

When $p_1 = p_2 = p$, every non-negative bundle satisfying

$$x_1 + x_2 = \frac{m}{p}$$

is optimal. Thus x_1 can be any quantity in $[0, m/p]$, but $x_2 = m/p - x_1$: the two quantities cannot be chosen independently.

For perfect complements, the consumer chooses $x_1 = x_2$. Substituting into the budget constraint gives

$$x_1^*(p_1, p_2, m) = x_2^*(p_1, p_2, m) = \frac{m}{p_1 + p_2}.$$

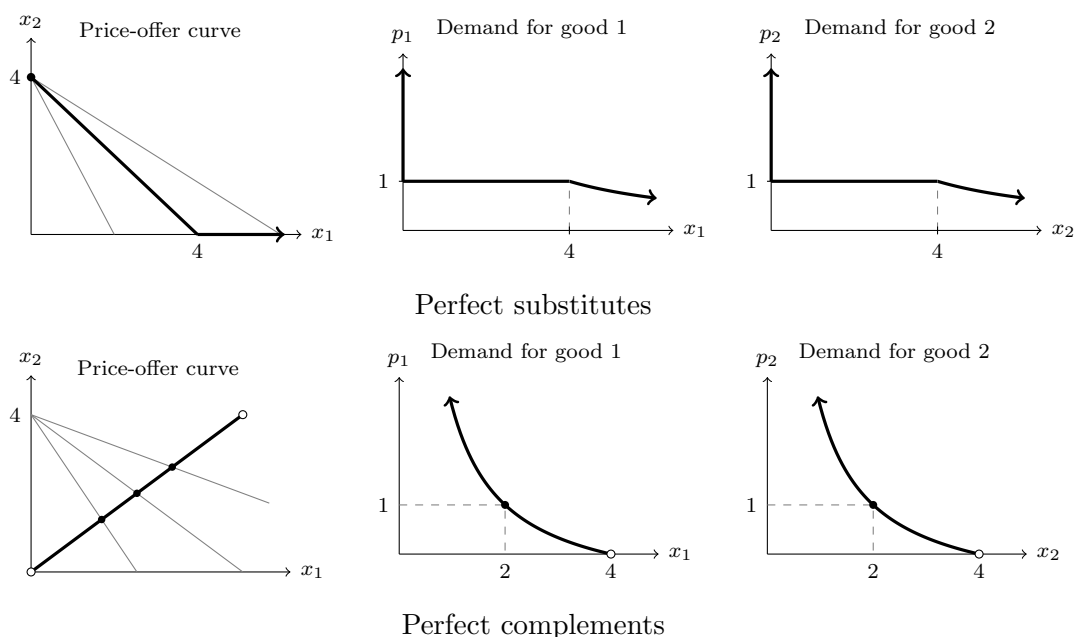
- (b) For perfect substitutes, each own-price demand has a vertical branch at zero quantity when its price exceeds the other price, a horizontal segment at equal prices, and a branch $x_i = m/p_i$ when its price is lower. With p_2 and m fixed, the p_1 price-offer curve consists of the point $(0, m/p_2)$, the entire segment $x_1 + x_2 = m/p_2$, and the horizontal ray $x_2 = 0, x_1 > m/p_2$.

For perfect complements, each own-price demand is downward-sloping. The p_1 price-offer curve is the segment

$$x_1 = x_2 = t, \quad 0 < t < \frac{m}{p_2}.$$

Its endpoints are limits as p_1 tends to infinity or zero. The limiting quantity on good 1's demand curve as $p_1 \rightarrow 0$ is m/p_2 .

The following diagrams use $m = 4$. For each own-price demand, the other price is fixed at 1; for each price-offer curve, $p_2 = 1$.



2. (a) Since utility is increasing in both goods, the budget is exhausted. Substituting $y = m - 2x$ into utility gives

$$U(x) = 2 \ln x + m - 2x, \quad 0 < x \leq m/2.$$

The unconstrained optimum satisfies $2/x - 2 = 0$, so $x = 1$. This is feasible only when $m \geq 2$. Hence the demands, and thus the Engel curves at the given prices, are

$$x^*(m) = \begin{cases} m/2, & 0 < m \leq 2, \\ 1, & m > 2, \end{cases} \quad y^*(m) = \begin{cases} 0, & 0 < m \leq 2, \\ m - 2, & m > 2. \end{cases}$$

With income on the vertical axis, the sports Engel curve is $m = 2x$ up to $(x, m) = (1, 2)$, followed by a vertical ray at $x = 1$. The cultural-activities Engel curve lies at $y = 0$ for $0 < m \leq 2$, followed by the line $m = y + 2$ for $m > 2$.

At general prices, the interior demands are

$$x^* = \frac{2p_y}{p_x}, \quad y^* = \frac{m}{p_y} - 2, \quad m > 2p_y.$$

Thus $\partial x^*/\partial m = 0$, while $\partial y^*/\partial m = 1/p_y > 0$. For small price changes that keep the relevant choices interior, the income effect on sports demand is zero. A price decrease gives a positive income effect on cultural demand; a price increase gives a negative one.

- (b) Initially, $m = 38$, $p_x = 2$ and $p_y = 1$, giving

$$(x^0, y^0) = (1, 36).$$

After p_y rises to 2, the Slutsky-adjusted budget is the cost of the original bundle at the new prices:

$$m^s = 38 + (2 - 1)36 = 74.$$

Thus a budget of £74,000 makes the original bundle just affordable. At the new prices and a budget b , the interior first-order conditions are

$$\frac{2}{x} = 2\lambda, \quad 1 = 2\lambda, \quad 2x + 2y = b,$$

giving $x = 2$ and $y = b/2 - 2$, provided $b > 4$. Therefore

$$(x^s, y^s) = (2, 35) \quad \text{at } b = 74, \quad (x', y') = (2, 17) \quad \text{at } b = 38.$$

The substitution effect compares the original and adjusted-budget choices; the income effect compares the adjusted-budget and final choices:

	Substitution	Income	Total
x	$2 - 1 = 1$	$2 - 2 = 0$	$2 - 1 = 1$
y	$35 - 36 = -1$	$17 - 35 = -18$	$17 - 36 = -19$

Sports demand rises by 1 through substitution alone. Cultural demand falls by 19: 1 through substitution and 18 through the reduction in purchasing power.

3. Let $p'_1 < p_1$ and let the original bundle be (x_1^0, x_2^0) . The Slutsky-adjusted income is

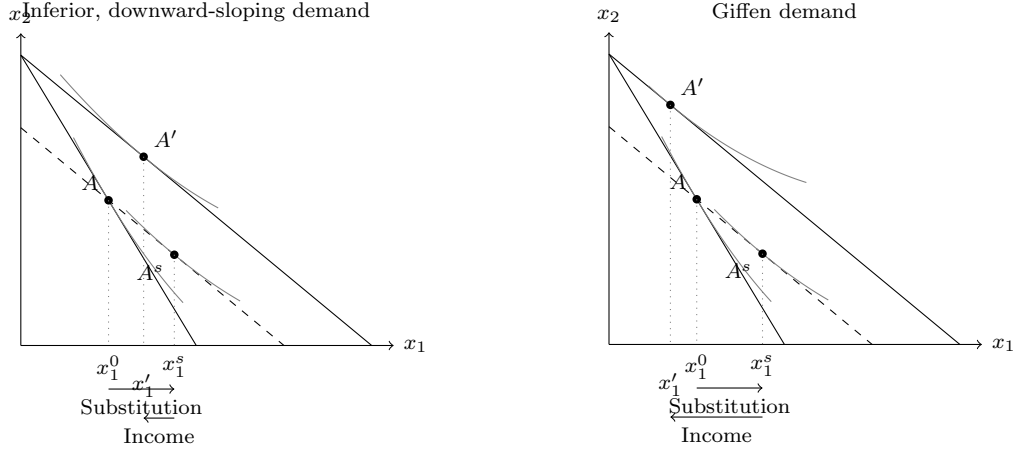
$$m^s = m + (p'_1 - p_1)x_1^0 < m.$$

Draw a budget line at the new prices passing through the original bundle A . Its optimal bundle is A^s . The final budget line is parallel to this adjusted line and farther out, since nominal income remains m ; its optimal bundle is A' .

The substitution effect raises demand for good 1, while the increase in purchasing power reduces demand for this inferior good:

$$\begin{aligned}\Delta x_1^s &= x_1(p'_1, p_2, m^s) - x_1(p_1, p_2, m) \geq 0, \\ \Delta x_1^m &= x_1(p'_1, p_2, m) - x_1(p'_1, p_2, m^s) < 0, \\ \Delta x_1 &= \Delta x_1^s + \Delta x_1^m.\end{aligned}$$

If the substitution effect is larger, $x_1^0 < x'_1 < x_1^s$ and demand slopes downwards. If the income effect is larger in absolute value, $x'_1 < x_1^0 \leq x_1^s$: this is the Giffen case. Equal magnitudes give no net change.



These diagrams are schematic. The dashed budget line passes through A and has the same slope as the final budget line. Unlike Hicks compensation, Slutsky compensation makes the original bundle affordable; it does not keep utility constant.

4. A rise in the price of a staple such as potatoes has two effects. The substitution effect reduces demand for the staple because it becomes more expensive relative to other goods. The loss of purchasing power increases demand for it if it is an inferior good: the household cuts back on more expensive foods and relies more heavily on the staple.

For Giffen behaviour, this positive income effect on the quantity demanded must outweigh the negative substitution effect. A large budget share makes the purchasing-power loss substantial, while limited close substitutes make the substitution effect relatively small. The household must also have some spending on other goods that it can cut back: a household already spending its entire budget on the staple must buy less when its price rises.

Being inferior is therefore necessary but not sufficient. This describes a possible mechanism; it does not establish that potatoes actually exhibited Giffen behaviour during the Irish famine.

5. (a) For $0 < p < 6$, Asian demand is $q^A = 12 - 2p$, so

$$\varepsilon^A(p) = \frac{p}{q^A} \frac{dq^A}{dp} = \frac{-2p}{12 - 2p} = -\frac{p}{6 - p}.$$

At $p = 3.5$, $q^A = 5$ and

$$\varepsilon^A(3.5) = -\frac{3.5}{2.5} = -1.4.$$

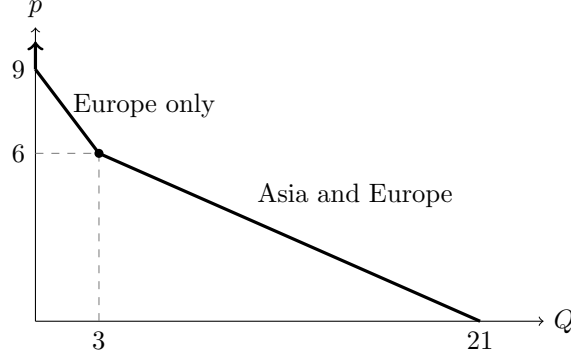
A small price increase lowers revenue because demand is elastic. Equivalently,

$$\frac{dR^A}{dp} = q^A(1 + \varepsilon^A) = 5(1 - 1.4) = -2 < 0.$$

- (b) Market demand is the horizontal sum of the two regional demands, each truncated at zero:

$$Q(p) = \begin{cases} 21 - 3p, & 0 \leq p < 6, \\ 9 - p, & 6 \leq p < 9, \\ 0, & p \geq 9. \end{cases}$$

Asian demand becomes zero at $p = 6$; European demand becomes zero at $p = 9$. The graph joins $(Q, p) = (0, 9)$, $(3, 6)$ and $(21, 0)$.



- (c) Asian inverse demand and revenue are

$$P^A(q) = 6 - \frac{q}{2}, \quad R^A(q) = P^A(q)q = 6q - \frac{q^2}{2}, \quad 0 \leq q \leq 12.$$

Hence marginal revenue is

$$MR^A(q) = \frac{dR^A}{dq} = 6 - q.$$

Revenue is maximised at $MR^A = 0$, giving

$$q^* = 6, \quad p^* = 3, \quad R^A(q^*) = 18.$$

Since $d^2R^A/dq^2 = -1 < 0$, this is the unique maximum. Demand has unit elasticity at this price.

6. (a) The Lagrangian is

$$\mathcal{L} = 2\sqrt{x_1} + x_2 + \lambda(m - p_1x_1 - p_2x_2).$$

The first-order conditions for an interior choice are

$$\frac{1}{\sqrt{x_1}} - \lambda p_1 = 0, \quad 1 - \lambda p_2 = 0, \quad m - p_1x_1 - p_2x_2 = 0.$$

Using $\lambda = 1/p_2$ gives

$$x_1^*(p_1, p_2, m) = \left(\frac{p_2}{p_1}\right)^2, \quad x_2^*(p_1, p_2, m) = \frac{m}{p_2} - \frac{p_2}{p_1}.$$

The assumption $m > p_2^2/p_1$ ensures $x_2^* > 0$. Utility is concave, so the first-order conditions identify the maximum.

- (b) Write $\varepsilon_{i,z} = (z/x_i^*)(\partial x_i^*/\partial z)$. The own-price elasticities are

$$\varepsilon_{1,p_1} = -2, \\ \varepsilon_{2,p_2} = -\frac{1}{x_2^*} \left(\frac{m}{p_2} + \frac{p_2}{p_1} \right) = -\frac{mp_1 + p_2^2}{mp_1 - p_2^2} < -1.$$

The cross-price elasticities are

$$\begin{aligned}\varepsilon_{1,p_2} &= 2, \\ \varepsilon_{2,p_1} &= \frac{1}{x_2^*} \frac{p_2}{p_1} = \frac{p_2^2}{mp_1 - p_2^2} > 0.\end{aligned}$$

The income elasticities are

$$\begin{aligned}\varepsilon_{1,m} &= 0, \\ \varepsilon_{2,m} &= \frac{1}{x_2^*} \frac{m}{p_2} = \frac{mp_1}{mp_1 - p_2^2} > 1.\end{aligned}$$

Both goods have downward-sloping, elastic demands; good 1 has constant own-price elasticity. The positive cross-price elasticities make the goods gross substitutes. Good 1 is independent of income in this interior range, so it is neither strictly normal nor inferior there. Good 2 is normal and has an income elasticity greater than one, making it a luxury in the technical sense.

The zero income effect for good 1 does not apply at all incomes. Below the threshold, $x_2 = 0$ and $x_1 = m/p_1$, so demand for good 1 does respond to income.

- (c) Rearranging the demand for good 1 gives its inverse demand:

$$p_1(x_1) = \frac{p_2}{\sqrt{x_1}}, \quad 0 < x_1 < \left(\frac{m}{p_2}\right)^2.$$

The upper bound ensures that good 2 is consumed in a positive amount. The marginal utilities and signed marginal rate of substitution are

$$MU_1 = \frac{1}{\sqrt{x_1}}, \quad MU_2 = 1, \quad MRS = \left. \frac{dx_2}{dx_1} \right|_u = -\frac{MU_1}{MU_2} = -\frac{1}{\sqrt{x_1}}.$$

At an interior optimum, $|MRS| = p_1/p_2$, so

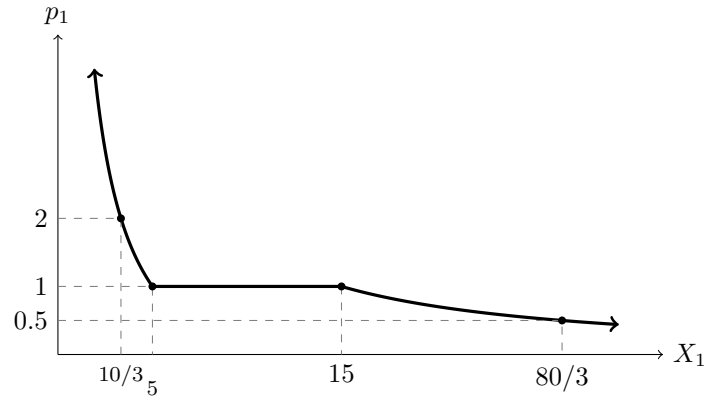
$$p_1 = p_2|MRS|.$$

The absolute MRS measures willingness to give up units of good 2 for an additional unit of good 1. Multiplying by p_2 expresses this marginal willingness to pay in monetary units. If $p_2 = 1$, inverse demand equals $|MRS|$.

7. (a) The perfect-complements consumer demands $10/(p_1+1)$ units of good 1. The perfect-substitutes consumer demands $10/p_1$ when $p_1 < 1$, zero when $p_1 > 1$, and any quantity in $[0, 10]$ at $p_1 = 1$. Adding these demands gives

$$X_1(p_1) = \begin{cases} \frac{10}{p_1} + \frac{10}{p_1+1} = \frac{10(1+2p_1)}{p_1(1+p_1)}, & 0 < p_1 < 1, \\ \frac{10}{p_1+1}, & p_1 > 1. \end{cases}$$

At $p_1 = 1$, the complements consumer demands 5, while the substitutes consumer can demand any quantity between 0 and 10. Thus the full range is $X_1(1) \in [5, 15]$.



- (b) At $p_1 = 2$ and $p_2 = 1$, the perfect-substitutes consumer buys only good 2, so demands zero units of good 1 both before and after the transfer. Initially, the perfect-complements consumer has income 10 and demands

$$x_1^C = \frac{10}{2+1} = \frac{10}{3}.$$

After the transfer, the two incomes are 8 for the substitutes consumer and 12 for the complements consumer. Market demand becomes

$$X'_1 = 0 + \frac{12}{2+1} = 4, \quad \Delta X_1 = 4 - \frac{10}{3} = \frac{2}{3}.$$

Total income remains £20, but the recipients have different demands. At these prices, each additional pound raises the complements consumer's demand for good 1 by $1/3$ of a unit, whereas changes in the substitutes consumer's income do not affect demand for good 1. Market demand therefore depends on the distribution of income, as well as total income.