

5SSPP217 Microeconomics
Week 3 exercises
Consumer demand

1. Consider the perfect substitutes utility function

$$U(x_1, x_2) = x_1 + x_2$$

and the perfect complements utility function

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

Let $p_1, p_2 > 0$ and $m > 0$, with $x_1, x_2 \geq 0$.

- (a) Derive the demands for both goods as functions of prices p_1, p_2 and income m . Include all optimal bundles when $p_1 = p_2$.
 - (b) For each utility function, draw the demand for each good as its own price varies, holding the other price and income fixed. Also draw the p_1 price-offer curve in (x_1, x_2) space, holding p_2 and m fixed. Include any sets of optimal bundles at equal prices.
2. A town allocates its budget between sports facilities (good x) and cultural activities (good y). Its citizens' preferences are represented by

$$U(x, y) = 2 \ln x + y, \quad x > 0, \quad y \geq 0.$$

Each unit of sports facilities costs £2000, and each unit of cultural activities costs £1000. Measure all budgets and prices in thousands of pounds, so that a budget m gives the constraint

$$2x + y \leq m.$$

- (a) Derive the Engel curves for both goods for all $m > 0$, identifying any corner solutions. For budgets at which both goods are consumed, what are the signs of the income effects on the demand for each good following a small increase or decrease in either price?
 - (b) Initially, the budget is £38,000. The cost of cultural activities then doubles to £2000 per unit, while the cost of sports facilities is unchanged. Find the original optimal bundle and the budget needed to afford that bundle at the new prices. Find the optimal bundle at the new prices with this adjusted budget, and with the original budget. Hence calculate the Slutsky substitution effect, income effect and total change in demand for each good.
3. In the lectures, we decomposed a price-induced change in demand into substitution and income effects for a normal good. Using the Slutsky decomposition, illustrate these effects for an inferior good following a decrease in its price. Put the inferior good on the horizontal axis, and hold income and the price of the other good fixed. Explain how the overall change depends on the relative magnitudes of the two effects, distinguishing the case of an inferior good with downward-sloping demand from the Giffen case.
4. Using potatoes as an example of a staple food, explain the conditions under which a price increase could lead a poor household to consume more of the staple. Discuss the relative strength of the substitution and income effects, the staple's share of the household's budget, and the availability of substitutes. Is being an inferior good sufficient for a good to exhibit Giffen behaviour?

5. For $p \geq 0$, the demands for widgets in Asia and Europe are

$$q^A(p) = \max\{12 - 2p, 0\}, \quad q^E(p) = \max\{9 - p, 0\}.$$

- (a) Show that the price elasticity of demand for widgets in Asia is -1.4 when $p = 3.5$. Would revenue in that market rise or fall following a small increase in the price?
 - (b) Suppose that widgets are not demanded anywhere else in the world. Derive world demand as a piecewise function of p and draw it, identifying the prices at which each regional demand becomes zero.
 - (c) Derive the inverse demand and marginal revenue functions for Asia. Find the price and quantity that maximise revenue in that market.
6. Consider a consumer with quasilinear utility

$$u(x_1, x_2) = 2\sqrt{x_1} + x_2, \quad x_1, x_2 \geq 0.$$

The consumer faces prices $p_1, p_2 > 0$ and has income m , where

$$m > \frac{p_2^2}{p_1}.$$

- (a) Using the Lagrange method, derive the demands $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$.
 - (b) Find the own-price, cross-price and income elasticities of both demands at prices and income satisfying the condition above. What do their signs tell you about whether the goods have downward-sloping demand, are gross substitutes or complements, and are normal, inferior or independent of income in this range?
 - (c) Derive the inverse demand for good 1 in the range where both goods are consumed, holding p_2 and m fixed. Explain its relationship to the marginal rate of substitution and marginal willingness to pay.
7. Consider an economy with two consumers. One has utility $U(x_1, x_2) = \min\{x_1, x_2\}$ and the other has utility $U(x_1, x_2) = x_1 + x_2$. Each consumer has income $m = \pounds 10$, and the price of good 2 is $p_2 = \pounds 1$ per unit. In the calculations below, express prices and income numerically in pounds.
- (a) Derive market demand for good 1 for all $p_1 > 0$ and draw it. Include the full range of possible market quantities when $p_1 = 1$.
 - (b) Suppose that $p_1 = 2$. Transfer $\pounds 2$ of income from the consumer with perfect substitutes preferences to the consumer with perfect complements preferences. Calculate market demand for good 1 before and after the transfer. Explain why market demand changes even though total income is unchanged.