

5SSPP217 Microeconomics
Week 2 solutions
Consumer choice

Quantities are continuously divisible and non-negative, unless stated otherwise. Budgets and prices are positive. We use the signed convention $MRS = dx_2/dx_1 = -MU_1/MU_2$.

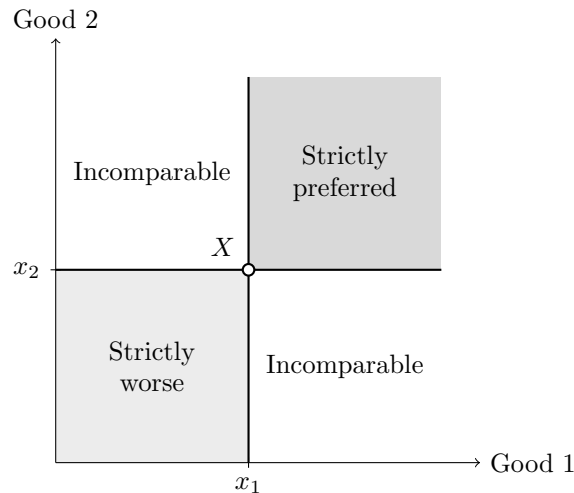
1. Wen's preferences

- (a) Write $X = (x_1, x_2)$ for the reference bundle and $Y = (y_1, y_2)$ for another bundle. Strict preference requires weak preference in one direction but not in the other. Hence the strictly preferred and strictly worse sets are, respectively,

$$S^+(X) = \{Y : y_1 \geq x_1, y_2 \geq x_2, Y \neq X\},$$

$$S^-(X) = \{Y : y_1 \leq x_1, y_2 \leq x_2, Y \neq X\}.$$

Both sets include their horizontal and vertical edges, except for X itself. The only bundle indifferent to X is X .



The open circle excludes X from both shaded sets. The diagram illustrates an interior reference bundle; at an axis, the same set definitions apply within the non-negative consumption space.

- (b) Wen's preferences are **not complete**. Bundles are incomparable when

$$y_1 < x_1 \text{ and } y_2 > x_2, \quad \text{or} \quad y_1 > x_1 \text{ and } y_2 < x_2.$$

For example, neither $(1, 2)$ nor $(2, 1)$ is weakly preferred to the other. Equality in either coordinate would instead allow a ranking.

Preferences are **monotonic**: increasing either good while holding the other fixed gives a strictly preferred bundle. More generally, a coordinatewise increase with at least one strict increase gives a strictly preferred bundle.

Preferences are **transitive**. If $Z \preceq Y$ and $Y \preceq X$, then, for $i = 1, 2$,

$$z_i \leq y_i \leq x_i.$$

Therefore $Z \preceq X$.

2. Angela's preferences

Bundle C is a convex combination of A and B :

$$\frac{1}{3}A + \frac{2}{3}B = \left(\frac{1}{3} \cdot 6 + \frac{2}{3} \cdot 18, \frac{1}{3} \cdot 12 + \frac{2}{3} \cdot 6 \right) = (14, 8) = C.$$

Since $A \sim B$, convexity implies

$$C \succeq A \quad \text{and} \quad C \succeq B.$$

Thus C is **weakly preferred** to both bundles. The information does not establish whether C is strictly preferred or indifferent to them. Strict convexity would imply strict preference, but it has not been assumed. For example, the utility function $u(x_1, x_2) = x_1 + 2x_2$ satisfies the stated assumptions and assigns utility 30 to all three bundles.

3. Scones and tea

Both goods raise utility, so the budget is exhausted. For $B > 0$ the optimal bundle is interior: a bundle with either good equal to zero has zero utility, whereas an affordable bundle with both goods positive has positive utility. The problem is

$$\max_{s, t \geq 0} s^{1/2} t^{1/3} \quad \text{subject to} \quad p_s s + t = B.$$

The Lagrangian is

$$\mathcal{L}(s, t, \lambda) = s^{1/2} t^{1/3} - \lambda(p_s s + t - B).$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial s} &= \frac{1}{2} s^{-1/2} t^{1/3} - \lambda p_s = 0, \\ \frac{\partial \mathcal{L}}{\partial t} &= \frac{1}{3} s^{1/2} t^{-2/3} - \lambda = 0, \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= B - p_s s - t = 0. \end{aligned}$$

Dividing the first two conditions gives

$$\frac{3t}{2s} = p_s, \quad p_s s = \frac{3}{2}t.$$

Substituting into the budget constraint yields $\frac{5}{2}t = B$. Therefore

$$\boxed{t^* = \frac{2B}{5}, \quad s^* = \frac{3B}{5p_s}.$$

Dr Pages spends two-fifths of his budget on tea and three-fifths on scones. These demands give the unique optimum. If $B = 0$, the only affordable bundle is $(s, t) = (0, 0)$.

4. Mr Richards

(a) The budget line is

$$4x + 15y = 150, \quad y = 10 - \frac{4}{15}x,$$

with intercepts $(37.5, 0)$ and $(0, 10)$. The indifference curve for utility k is

$$2x + 5y = k, \quad y = \frac{k}{5} - \frac{2}{5}x,$$

restricted to the non-negative quadrant.

Substituting the budget line into utility gives

$$U(x) = 2x + 5 \left(10 - \frac{4}{15}x \right) = 50 + \frac{2}{3}x, \quad 0 \leq x \leq 37.5.$$

Utility increases with x , so there is a corner solution: all income is spent on good x . Equivalently,

$$|\text{MRS}| = \frac{2}{5} > \frac{4}{15} = \frac{p_x}{p_y}.$$

Therefore

$$(x^*, y^*) = (37.5, 0), \quad U(x^*, y^*) = 75.$$

The number 75 is the value of the specified utility function; it is not a cardinal measure of happiness.

(b) With free membership, the new budget line is

$$4x + 10y = 150, \quad y = 15 - \frac{2}{5}x.$$

It pivots outwards around $(37.5, 0)$: the y -intercept rises from 10 to 15, and the slope changes from $-4/15$ to $-2/5$.

On the new budget line,

$$U(x) = 2x + 5 \left(15 - \frac{2}{5}x \right) = 75.$$

Every bundle on this line is optimal. The full set is

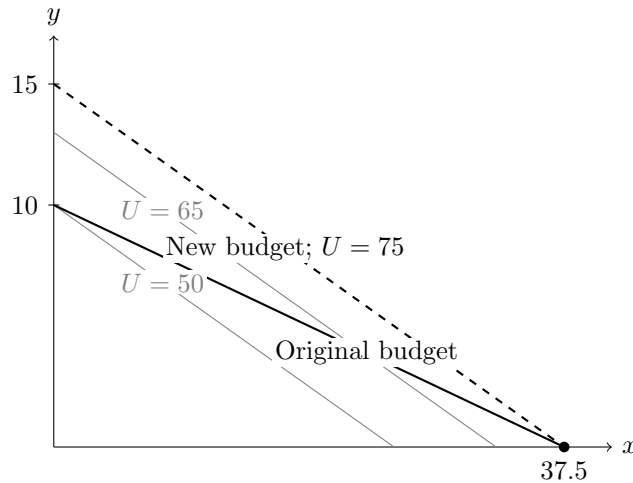
$$\left\{ \left(x, 15 - \frac{2}{5}x \right) : 0 \leq x \leq 37.5 \right\}.$$

The new budget line coincides with the indifference curve $U = 75$.

A membership fee $f > 0$ would reduce the available budget to $150 - f$. For $0 < f \leq 150$, the maximum utility at the discounted prices would then be

$$\frac{150 - f}{2} = 75 - \frac{f}{2} < 75.$$

Hence he would not pay a positive fee: his maximum willingness to pay is zero.



5. Sports facilities and cultural activities

- (a) For an arbitrary utility level k , the indifference curve is

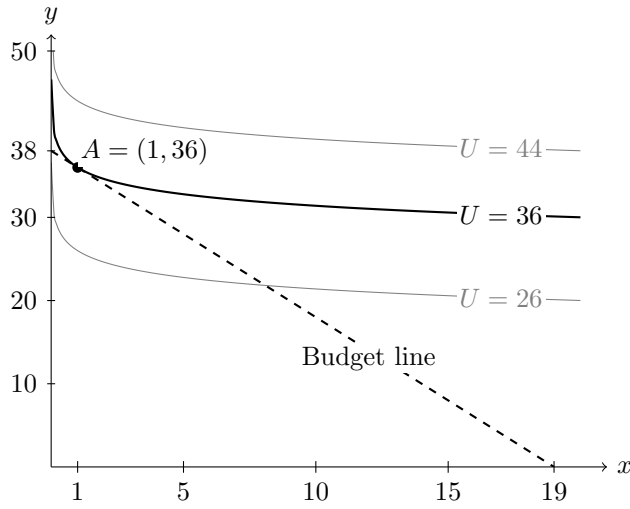
$$2 \ln x + y = k, \quad y = k - 2 \ln x,$$

with $0 < x \leq e^{k/2}$ so that $y \geq 0$. Preferences are quasilinear. Each curve has slope $-2/x$ and second derivative $2/x^2 > 0$.

Measuring expenditure in thousands of pounds, the budget line for $m = 38$ is

$$2x + y = 38, \quad y = 38 - 2x.$$

Its intercepts are $(19, 0)$ and $(0, 38)$, although utility is not defined at $x = 0$. The graph shows three indifference curves and the optimum $A = (1, 36)$.



- (b) For $m > 2$, the problem is

$$\max_{x>0, y\geq 0} 2 \ln x + y \quad \text{subject to} \quad 2x + y = m.$$

The Lagrangian is

$$\mathcal{L}(x, y, \lambda) = 2 \ln x + y - \lambda(2x + y - m).$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x} &= \frac{2}{x} - 2\lambda = 0, \\ \frac{\partial \mathcal{L}}{\partial y} &= 1 - \lambda = 0, \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= m - 2x - y = 0. \end{aligned}$$

Hence $\lambda = 1$, $x^* = 1$ and $y^* = m - 2$:

$$\boxed{x^* = 1, \quad y^* = m - 2.}$$

Since $m > 2$, this solution is feasible and interior. On the budget line, utility is $2 \ln x + m - 2x$, whose second derivative is $-2/x^2 < 0$, so it is the unique maximum.

(c) With budget $2m$, the same calculation gives

$$\boxed{x^* = 1, \quad y^* = 2m - 2.}$$

Sports provision remains constant, while cultural provision rises by m . In this interior region, additional income is spent entirely on the good that enters utility linearly.

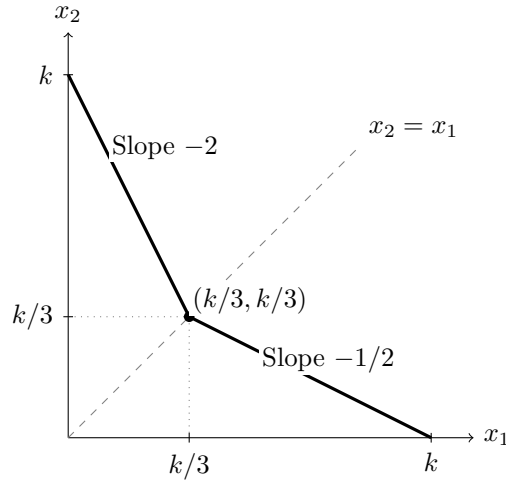
The income-invariance statement is conditional on sufficient income. For completeness, if $0 < m \leq 2$, the optimum is instead $(x^*, y^*) = (m/2, 0)$. Thus sports demand does depend on income at low budgets; it becomes constant once m reaches 2.

6. Kinked preferences

First draw an indifference curve for utility $k > 0$. If $x_1 > x_2$, then $x_1 + 2x_2 < 2x_1 + x_2$, so the curve is given by $x_1 + 2x_2 = k$. If $x_1 < x_2$, the curve is given by $2x_1 + x_2 = k$. At $x_1 = x_2$ the two expressions coincide. Consequently, the full indifference curve is

$$x_2 = \begin{cases} k - 2x_1, & 0 \leq x_1 \leq k/3, \\ (k - x_1)/2, & k/3 \leq x_1 \leq k. \end{cases}$$

Its intercepts are $(0, k)$ and $(k, 0)$, and its kink is at $(k/3, k/3)$. The MRS is -2 on the upper segment and $-1/2$ on the lower segment; it is not defined at the kink.



Both goods raise utility, so the consumer exhausts the budget:

$$p_1 x_1 + p_2 x_2 = m.$$

Write $r = p_1/p_2$ for the relative price and $d = m/(p_1 + p_2)$ for the quantity of each good at the affordable diagonal bundle.

- If $r > 2$, good 2 offers more utility per pound on both segments. The unique optimum is $(0, m/p_2)$.
- If $r < 1/2$, good 1 offers more utility per pound on both segments. The unique optimum is $(m/p_1, 0)$.
- If $1/2 < r < 2$, moving towards the diagonal along the budget line raises utility from either side. The optimum is therefore at the kink, where

$$x_1 + 2x_2 = 2x_1 + x_2, \quad x_1 = x_2.$$

Combining this with the budget constraint gives the unique optimum (d, d) .

- If $r = 2$, the budget line has slope -2 and coincides with the upper segment of the highest affordable indifference curve. Every budget-line bundle with $x_1 \leq x_2$ is optimal: the segment from $(0, m/p_2)$ to (d, d) .
- If $r = 1/2$, the budget line has slope $-1/2$ and coincides with the lower segment of the highest affordable indifference curve. Every budget-line bundle with $x_1 \geq x_2$ is optimal: the segment from (d, d) to $(m/p_1, 0)$.

The full set of optimal bundles is therefore

$$\mathcal{D}(p_1, p_2, m) = \begin{cases} \{(m/p_1, 0)\}, & r < 1/2, \\ \left\{ \left(z, \frac{m - p_1 z}{p_2} \right) : d \leq z \leq \frac{m}{p_1} \right\}, & r = 1/2, \\ \{(d, d)\}, & 1/2 < r < 2, \\ \left\{ \left(z, \frac{m - p_1 z}{p_2} \right) : 0 \leq z \leq d \right\}, & r = 2, \\ \{(0, m/p_2)\}, & r > 2. \end{cases}$$

These are not perfect-complements preferences: both segments slope downwards, so substitution is possible along each segment. For $1/2 < r < 2$, the optimal bundle happens to coincide with the optimum under $\min\{x_1, x_2\}$, but this does not hold for all prices. If $m = 0$, the only affordable bundle is $(0, 0)$.