

5SSPP217 Microeconomics
Week 1 exercises
Maths refresher

1. A firm selling beds is a price taker. The current market price of beds is £200. The firm's total costs are given by the equation

$$C(q) = 100 + 160q + 0.5q^2,$$

where q is the number of beds produced (and sold).

- (a) What is the firm's optimal production level? How much profit does it earn?
 - (b) Suppose that fixed costs increase from their original level of 100 to 1000. How does this affect the firm's optimal decision?
2. Recall that the k -level curve of the two-variable function $f(x, y)$ is the set of all points in the domain of f at which f takes on the value k , i.e. such that $f(x, y) = k$. Find the mathematical expression for the k -level curve and draw a map of level curves for several values of k corresponding to the functions below:

- (a) $f(x, y) = x + y$
- (b) $f(x, y) = xy$
- (c) $f(x, y) = \sqrt{x} + y$
- (d) $f(x, y) = x^2 + y^2$

3. Let $a, b > 0$, $p_1, p_2 > 0$, and $m > 0$. Maximise

$$U(x_1, x_2) = x_1^a x_2^b$$

subject to the constraint

$$p_1 x_1 + p_2 x_2 = m$$

using the Lagrange method. Derive the consumer's optimal demands x_1^* and x_2^* .

4. Suppose that a car manufacturer produces its output Y by combining labour L and capital K according to the technology

$$Y = (K^{0.5} + L^{0.5})^2.$$

The production target of the company is $Y = 1000$ cars. The company wants to produce that level of output at the minimum possible cost. The wage rate is w and the rental rate of capital is r , so the firm's production cost is

$$wL + rK.$$

- (a) Derive the first-order conditions using the Lagrange method.
- (b) Show that the firm's demand for labour and capital are given by

$$L^*(w, r) = \frac{1000}{\left(1 + \frac{w}{r}\right)^2},$$
$$K^*(w, r) = \frac{1000}{\left(1 + \frac{r}{w}\right)^2}.$$

5. Suppose that a firm using capital K and labour L has the following production function:

$$F(K, L) = AK^\alpha L^\beta,$$

with $A, \alpha, \beta > 0$.

- (a) Find the marginal product of labour and the marginal product of capital. Show that both are positive. Show that the marginal products are diminishing if and only if $0 < \alpha < 1$ and $0 < \beta < 1$.
- (b) What is the degree of homogeneity of this production function? Recall that a function $f(x, y)$ is homogeneous of degree k if

$$f(tx, ty) = t^k f(x, y)$$

for any $t > 0$.